Trivalent Exh and summative predicates*

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Abstract Many expressions display 'homogeneity': they quantify as universals in positive sentences, but as negated existentials in negative sentences. This paper aims to partly rethink work claiming that homogeneity is the result of exhaustification. I focus on 'summative' predicates like colour adjectives, and the claim that they are universal in positive sentences because they exclude one another. Three puzzles arise on this approach: the existence of truth-value gaps in non-homogeneous situations, the existence of non-maximality, and contrasts between sentential negation and other downward-entailing environments. I show that all of these difficulties can be resolved if exhaustification is trivalent rather than bivalent.

Keywords: exhaustivity, predicates, colour terms, homogeneity, non-maximality, plural predication

1 Introduction

Quantification by predicates is often subject to an all-or-nothing or 'homogeneity' effect; the positive predication is universal while the negative is a negated existential:

(1) a. The square is blue. ⤳ 'All of the square is blue'
     b. The square isn't blue. ⤳ 'None of the square is blue'

Predicates like *blue* that are true of an individual by virtue of being true of its parts are called 'summative,' as opposed to 'integrative' predicates like *square* that are true of an individual as an atom (Löbner 2000). Identical quantificational patterns are observed beyond summative predicates (Križ 2015), including in plural predication:

(2) a. The children sang. ⤳ 'All of the children sang'
     b. The children didn't sing. ⤳ 'None of the children sang'

However, this quantification is not robust for either summative predicates or plural predication. Weaker meanings are often observed; this has been called 'non-maximality':

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(3) a. This leaf is orange. (felicitous if the leaf is e.g. 80% orange)

b. The students cheered. (felicitous if e.g. 80% of the students cheered)

Some work has taken the paradigm in (1) and (2) to involve exhaustification in positive sentences (1a)/(2a) but not negative ones (1b)/(2b) (Magri 2014; Bar-Lev 2021; Paillé 2022a). For these theories, at least some of the following empirical observations about homogeneity are not immediately predicted:

i. homogeneity effects are usually described as giving rise to truth-value gaps;

ii. they can be non-maximal; and

iii. the presence or absence of exhaustification is regulated by whether the environment of the predication is positive or negative, and not whether it is upward- or downward-entailing (as is normally the case for exhaustification).

More specifically, none of these observations are predicted for the exhaustification-based approach in Paillé 2022a,[1] which I focus on in this paper. I will suggest to overcome these shortcomings by claiming that exhaustification is trivalent, being computed by a trivalent operator called 'Pexh' (Bassi, Del Pinal & Sauerland 2021):

(4) $[\![\text{Pexh}_{\text{ALT}}(p)]\!] = \begin{cases} 1, \text{ iff } [\![p]\!] = 1 \land \forall q \in \text{ALT}([\![q]\!] = 1 \to [\![p]\!] \subseteq [\![q]\!]); \\ 0, \text{ iff } [\![p]\!] = 0; \\ \#, \text{ otherwise} \end{cases}$

This paper is organized as follows. Section 2 presents what I will call the Exclusion theory of summative predicates (Paillé 2022a), as well as an exhaustification-based approach to plural predication from Bar-Lev (2021). Section 3 then discusses the three problems listed above for the Exclusion theory, and shows how adopting Pexh can solve them. Section 4 discusses how the theoretical suggestions made for summative predicates in this paper impact our understanding of plural predication, and section 5 concludes.

2 Two exhaustification accounts of homogeneity

In this section, I outline the Exclusion theory for summative predicates (section 2.1) and Bar-Lev's (2021) 'Inclusion' theory of plural predication (section 2.2). Both involve exhaustification modelled through the Exh(aust) operator of Chierchia, Fox & Spector (2012) (though Bar-Lev modifies it, as we will see):

(5) $[\![\text{Exh}_{\text{ALT}}(p)]\!] = 1$ iff $[\![p]\!] = 1$ and $\forall q \in \text{ALT} ([\![q]\!] = 1 \to [\![p]\!] \subseteq [\![q]\!])$.

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1 I am ignoring chapter 6 of that work, which overlaps in part with this paper.

Exh asserts that its prejacent is true and that the non-entailed alternatives are false. Thus, falsity results if p is false or if a non-entailed alternative is true.

After outlining these theories, I end the section (section 2.3) by showing that exhaustification-based theories of homogeneity must involve some locality constraint on Exh.

2.1 The Exclusion theory of summative predicates

(6) repeats from (1) the basic quantificational pattern of colour terms:

(6)   a. The square is blue. ⤳ 'All of the square is blue'

       b. The square isn't blue. ⤳ 'None of the square is blue'

Harnish (1976) and Levinson (1983) argue summative predicates are lexically weak:

(7) $[\![\text{blue}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \text{blue}(y)].$ (abbreviated as '$\lambda x.\text{blue}_{\exists}(x)$')

The truth conditions of negative sentences (6b) with these predicates follow immediately. As for the universal quantification in positive sentences (6a), it must be the result of some strengthening mechanism. On the Exclusion theory, this is obtained by taking summative predicates to exclude other same-class predicates. For instance, colour terms exclude other colour terms:

(8)   a. Exh_{ALT} [the square is blue].

       b. ALT = {the square is blue$_{\exists}$, the square is white$_{\exists}$, the square is red$_{\exists}$, ... }

       c. $[\![(8a)]\!] = 1$ iff $\text{blue}_{\exists}(s) \wedge \neg\text{white}_{\exists}(s) \wedge \neg\text{red}_{\exists}(s) \wedge \neg \ldots$

If the square is at least partly blue and has no other colour, it must be entirely blue.

2.2 The Inclusion theory of plural predication

Plural predication is in many ways similar to summative predicates, motivating some unification of analysis; (9) is repeated from (2).

(9)   a. The children sang. ⤳ 'All of the children sang'

       b. The children didn't sing. ⤳ 'None of the children sang'

Exhaustification-based approaches to plural predication start with the claim that plural predication is lexically existential; prior to being strengthened, (9a) is true as long as one of the children sang. To strengthen this, however, the Exclusion theory described above is of no use (Paillé 2022b). If we applied the logic of the Exclusion theory to (9), we would try to strengthen (9a) by excluding alternatives obtained by

replacing *sing* with conceptually related predicates—perhaps *dance* or *speak*. But this does not lead to universal meaning, and what is more, it creates entailments that are not intuited without contrastive focus on *sang*—nothing in (9a) suggests that the children are not dancing, for example.

In contrast, Bar-Lev (2021) derives universal meaning for (9a) by claiming (i) it carries subdomain alternatives, and (ii) Exh asserts that these subdomain alternatives are true. If there are two children, *a* and *b*, the basic meaning is that *a* or *b* sang; the alternatives are '*a* sang' and '*b* sang'; Exh cannot consistently assert that both of these alternatives are false (Fox 2007), but it can consistently assert that they are both true (see Bar-Lev 2021 for a definition of Exh letting it assert that non-excluded alternatives are true). The resulting meaning is that all the children sang.

The fact that the Exclusion theory cannot be extended to plural predication should not be viewed as a problem for it. There is nothing wrong with viewing different homogeneity effects as only partly theoretically unified: they can all be due to exhaustification, but involving different kinds of alternatives. More importantly, attempting to extend theories of plural predication to summative predication does not successfully capture all the data with summative predicates (Paillé 2022a: ch. 5): there is currently no competitor to the Exclusion theory for summative predicates that could capture all homogeneity effects in one go.

2.3 Local Exh with summative and plural predication

Putting aside negative sentences for now, a salient property of the computation of universal meaning for summative and plural predication is that it must be computed locally (see ch. 2 of Paillé 2022a). For instance, as shown in (10a), summative predicates are as strong under *if* as in (6a); the same goes for plurals (10b).

(10) a. If the square is blue, jump five times. ⤳ 'If the square is entirely blue, ...'

  b. If you solve the problems, you will pass the exam. (Križ 2015: 27)
  ⤳ 'If you solve all the problems, you will pass the exam'

On the Exclusion account, to capture the universal meanings in (10), one has to claim that Exh must appear below *if*:

(11) ⟦If [Exh$_{\text{ALT}}$ the square is blue], jump five times⟧
  = 1 iff you should jump five times if the square is blue$_\exists$ and no other colour.

This is striking given that exhaustification is usually dispreferred in downward-entailing (DE) environments; it leads to global weakening. Yet universal meanings under *if* seem to be the default. To be sure, non-universal meanings exist too (12), but such meanings should be analyzed as instances of non-maximality independent of the environment's downward-entailingness:

(12)   If you touch the statues, you will be asked to leave.   (Križ 2015: 27)
        ⇝ 'If you touch any of the statues, you will be asked to leave'

That is, (12) should not be taken to show that Exh is fully flexible in whether it is present below *if*. If Exh was fully flexible—if this was a normal exhaustification effect—it would be puzzling that the sentences in (10) are preferrably intuited as having universal quantification below *if*, since having Exh below a DE operator is normally dispreferred. Rather, (12) should be taken to involve a local Exh together with non-maximality (see sections 3.2 and 4).

Further, this locality constraint is not special to DE environments. Consider (13).

(13)   Exactly one square is blue. ⇝ 'Exactly one square is entirely blue.'

An exclusive meaning for *blue* cannot be captured with a global Exh (14a). I assume there are two alternative-triggering expression in (13), namely *one* and *blue*, so the alternatives for a global Exh include all the sentences obtained by replacing one or both of these lexical expressions (Sauerland 2004). For simplicity, pretend the only numerals are zero to two and the only colours are blue, red, and white.

(14)   a. Exh$_{ALT}$ [exactly one square is blue].

        b. ALT = $\left\{ \begin{array}{lll} \text{ex. 0 sq. are blue,} & \text{ex. 1 sq. is blue,} & \text{ex. 2 sq. are blue,} \\ \text{ex. 0 sq. are red,} & \text{ex. 1 sq. is red,} & \text{ex. 2 sq. are red,} \\ \text{ex. 0 sq. are white,} & \text{ex. 1 sq. is white,} & \text{ex. 2 sq. are white} \end{array} \right\}$

Which of these alternatives are innocently excludable (Fox 2007)? In fact, very few of them. None of the alternatives involving other colour terms are excludable: excluding them would result in the entailment that there is no numeral n such that n squares are partly red or partly white. Thus, only *exactly zero squares are blue* and *exactly two squares are blue* are excludable; but the exclusion of these alternatives is vacuous, because their falsity is already entailed by Exh's prejacent. Thus, a global Exh (14a) leads to no strengthening.

On the other hand, the right meaning can be obtained if Exh is constrained to being local with summative predicates. The nature of this locality constraint is not the focus of this paper; in (15), I simply assume a non-propositional Exh that takes a predicate and excludes non-entailed predicates based on a generalized notion of entailment (but I will write Exh as adjoining to clauses in the rest of the paper).

(15)   a. Exactly one square is [Exh$_{ALT}$ blue].

        b. ALT = {blue, red, white}

        c. ⟦(15a)⟧ = 1 iff exactly one square is (blue$_\exists$ & not red$_\exists$ & not white$_\exists$).

Whatever its exact nature, the locality constraint must be stricter than Exh simply needing to be in the same clause as the predicate: (13) is monoclausal, but a global Exh must still be ruled out.

3 Three problems for the Exclusion theory, and how Pexh solves them

As it stands, the Exclusion theory of summative predicates faces three difficulties, which I take one by one in the following subsections.[2] These difficulties can all be overcome by replacing the Exh operator defined in (5) with the trivalent Exh proposed by Bassi et al. (2021). Trivalent Exh (henceforth 'Pexh') leads to the same truth conditions as bivalent Exh, but defines non-complementary falsity conditions:

$$(16) \quad [\![\text{Pexh}_{\text{ALT}}(p)]\!] = \begin{cases} 1, \text{ iff } [\![p]\!] = 1 \wedge \forall q \in \text{ALT}([\![q]\!] = 1 \rightarrow [\![p]\!] \subseteq [\![q]\!]); \\ 0, \text{ iff } [\![p]\!] = 0; \\ \#, \text{ otherwise} \end{cases}$$

Specifically, Pexh leads to a sentence being neither true nor false if the prejacent p is true but there is also a non-entailed alternative that is true.

3.1 Problem 1: Truth-value gaps

Löbner (2000) suggests that the falsity conditions of a sentence p are the same as the truth conditions of its negation $\neg p$. Thus, given that truth conditions of positive and negative summative predications are not complementary, it must be that both positive and negative summative predications involve truth-value gaps (cf. Križ 2015):

$$(17) \quad [\![\text{The square is blue}]\!] = \begin{cases} 1, \text{ iff the square is all blue;} \\ 0, \text{ iff the square is not blue at all;} \\ \#, \text{ otherwise} \end{cases}$$

Call this a 'heterogeneity-gap.' As stated, the Exclusion account does not predict heterogeneity-gaps; *the square is blue* would be true if the square is exclusively blue, and false otherwise. Further, heterogeneity-gaps cannot be obtained by a change to summative predicates' lexical meanings. Attempting to derive the gaps from lexical meaning would involve positing an all-or-nothing presupposition à la Löbner (2000):

$$(18) \quad [\![\text{blue}]\!] = \lambda x : \forall y \sqsubseteq x[\text{blue}(y)] \vee \neg \exists y \sqsubseteq x[\text{blue}(y)]. \ \forall y \sqsubseteq x[\text{blue}(y)].$$

The problem with (18) is that summative predicates are weaker than universal in certain positive sentences, including ones with conjunction (see Paillé 2022a for discussion, including arguments against analysing (19) in terms of non-maximality):

(19)   The square is blue and orange.

But with an all-or-nothing presupposition, it is impossible to have colour terms be defined at all if they are not universal or negated existentials.

    On the other hand, if Exh is trivalent as proposed by Bassi et al. (2021), the Exclusion theory effortlessly predicts heterogeneity-gaps to arise:

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2 Some of these puzzles (Problems 1 and 3, but not Problem 2) also arise for the Inclusion theory.

(20) $[\![\text{Pexh}_{\text{ALT}} \text{ [the square is blue]}]\!]$

$$= \begin{cases} 1, \text{ iff blue}_\exists(s) \wedge \neg\text{white}_\exists(s) \wedge \neg\text{red}_\exists(s) \wedge \neg \ldots; \\ 0, \text{ iff } \neg\text{blue}_\exists(s); \\ \#, \text{ otherwise} \end{cases}$$

Consider the value predicted if the square is half blue, half white. The sentence cannot be true, because $\neg\text{white}_\exists(s)$ does not hold. The sentence also cannot be false, because $\neg\text{blue}_\exists(s)$ does not hold. Thus, the sentence is neither true nor false.

3.2 Problem 2: Non-maximality

Non-maximality is observed in both positive (21a) and negative (21b) summative predications:

(21)   a. i. SCENARIO: *Visitors at a bullfighting arena are not permitted to wear any red, but my shirt is half red, half white. A security guard says:*

   ii. You can't go in, your shirt is red.

   b. i. SCENARIO: *Tennis courts in a rich neighbourhood only allow people in if they are dressed exclusively in white. You try to go to a tennis court with a half white, half red shirt. A security guard says:*

   ii. You can't go in, your shirt isn't white.

In (21a-ii), *red* is existential in a positive sentence; in (21b-ii), *white* is a negated universal. Call the summative predications in these examples 'non-maximal' and 'non-minimal' respectively (with the pattern as a whole also called 'non-maximality').

Nothing in the Exclusion theory currently predicts non-maximality; (21a-ii), for instance, is predicted to exclude the shirt having any colour other than red. To fix this issue, I will turn to Križ's (2015) theory of non-maximality, which derives non-maximality from heterogeneity-gaps. Then, I will compare this approach to an alternative route inspired by Bar-Lev (2021), which does not require heterogeneity-gaps (and therefore does not require Pexh), to see that this route to non-maximality does not lead to the right results when used in the Exclusion theory.

3.2.1   A theory of non-maximality requiring Pexh

Križ (2015) starts by observing that in positive sentences, with both plurals and summative predicates, non-maximality and heterogeneity-gaps disappear with *all*:

(22)   a. All the professors smiled.

   b. All of the shirt is red.

This suggests a connection between the two. Križ also relies on the standard assumption (e.g. van Rooij 2003) that a QUD partitions worlds by how they resolve it. Let's consider the sentence (23) with a toy model of three worlds (24) involving different amounts of red on the shirt:

(23) The shirt is red.

(24) $\left\{ \begin{array}{ll} w_1: & \text{the shirt is all red,} \\ w_2: & \text{the shirt is half red,} \\ w_3: & \text{the shirt is not red at all} \end{array} \right\}$

If the QUD is 'How much red does the shirt have?' or 'What does the shirt look like?,' all of these worlds are in their own cell; they all correspond to different answers to the QUD. But if the QUD is 'Does the shirt have any red on it?,' w_1 and w_2 both correspond to the answer 'yes,' so they are in the same cell.

From here, Križ (2015) suggests to take to the letter Grice's (1975: 75) maxim of Quality, which Grice phrased as "Do not say what you believe to be false" rather than "Say what you believe to be true"—these are not equivalent in a trivalent semantics. Križ (2015) suggests that speakers can utter sentences that are neither true nor false, as long as in the given QUD, the world of utterance is in the same cell as the worlds in which the sentence is true. In our toy scenario, (23) is only true in w_1 regardless of the QUD; but with a QUD putting w_2 in the same cell as w_1, (23) can be uttered in w_2. Thus, (23) is sometimes felicitous even if the shirt is not all red.

3.2.2 Attempting to obtain non-maximality without Pexh

An alternative route to non-maximality would be to prune alternatives; this is how Bar-Lev (2021) suggests to obtain it for the Inclusion theory. If non-maximality can be obtained by pruning alternatives, non-maximality would no longer require Križ's theory (and therefore Pexh)—although the adoption of Pexh would still be motivated by Problem 1 (section 3.1) and Problem 3 (section 3.3).

For the Exclusion theory, obtaining non-maximality by pruning alternatives would mean not negating alternatives with certain colour terms. For (25a) in the bull-fighting scenario in (21a-i), the claim would be that red is the only relevant colour for the security guard's purposes, and therefore all other alternatives are pruned—as if there was no exhaustification at all:

(25) a. Your shirt is red.

b. ALT $=$ {Your shirt is red, ~~Your shirt is white~~, ~~Your shirt is blue~~, ...}

c. $[\![\text{Exh}_{\text{ALT}} (25a)]\!] = 1$ iff your shirt is red$_\exists$.

This result is in line with the intuited meaning of (25a). But non-maximality is often stronger than existential:

(26)   The leaf is orange.

In many situations, (26) can only be uttered of a leaf that is at least mostly orange. A scenario can bring this out explicitly:

(27)   SCENARIO: *For an art installation, you are making a large mosaic using leaves. There is a part of the drawing that should all be solid orange, but this part is still missing a lot of leaves. People will be looking at the mosaic from a distance to appreciate the full view of it, so it does not matter if the leaves you find are not actually fully orange, as long as they are mostly orange.*

(26) in scenario (27) is clearly acceptable if uttered of a leaf that is mostly or entirely orange, while being infelicitous about a leaf with large amounts of other colours.

For the Exclusion theory of summative predicates, the problem for the pruning route to non-maximality is that it is in fact impossible to obtain stronger-than-existential non-maximality. As soon as a single alternative is pruned, the asserted colour adjective can only be existential. This is shown in (28), which (for the sake of illustration) prunes some but not all alternative colour terms.

(28)   a. ALT = {The leaf is orange$_\exists$, The leaf is pink$_\exists$, ~~The leaf is green$_\exists$~~, ~~The leaf is brown$_\exists$~~, The leaf is blue$_\exists$, … }

        b. $[\![\text{Exh}_{\text{ALT}}\,(26)]\!] = 1$ iff the leaf is orange$_\exists$, maybe green$_\exists$ or brown$_\exists$, and no other colour.

The truth conditions in (28b) only require the leaf to have an orange part, however small; the leaf might be mostly green or brown. The assertion is strengthened in that the leaf cannot have any pink or blue on it, but *orange* remains existential.

In contrast, Križ's approach can distinguish between many different quantifica-tional forces, predicting that non-maximality can be stronger than existential. Since the QUD in scenario (27) is 'Is the leaf at least mostly orange?,' worlds in which the leaf is fully or mostly orange are in one cell, while worlds in which the leaf is less than mostly orange are in another cell. Thus, (26) is semantically neither true nor false, but felicitous if the leaf is mostly orange; and it is infelicitous if the leaf is less than mostly orange.

The picture emerging so far involves some back-and-forth. Colour terms are existential; in positive sentences, they are semantically strengthened to be exclusive of other colour terms; this strong meaning can then be weakened again in the pragmatics, due to the heterogeneity-gap. Thus, the meaning of non-maximal summative predicates involves strengthening followed by weakening. Why not

claim instead that colour terms are semantically weak (or ambiguous: Krifka 1996) and strengthened when they are *not* non-maximal? The reason for this is that there is more than just pragmatics to the strength of colour terms. Consider cases where colour terms are given a universal interpretation even though it results in a contradiction:

(29)   a. #The blue orange square is large.

        b. #Some blue squares are orange.

This universal meaning could not come out of the pragmatics, precisely because the pragmatic component would not create contradictory meaning out of consistent lexical material. See Paillé 2022a for more discussion motivating a semantic analysis of the strong meaning of summative predicates.

    In sum, to obtain non-maximality with colour terms, I suggest to adopt Pexh and Križ's theory of non-maximality.

3.3   Problem 3: Negation vs. other DE environments

In section 2, we observed an odd fact about summative predicates: they are weak under *not*, but strong in other DE environments:

(30)   a. The square is not blue.   (*blue* is weak under *not*)

        b. If the square is blue, jump five times.   (*blue* is strong under *if*)
           ⤳ 'If the square is entirely blue, …'

We also saw that, on the Exclusion account, capturing the universal meaning of *blue* in (30b) involves claiming that Exh must appear below *if* in (30b):

(31)   ⟦If [Exh$_{\mathrm{ALT}}$ the square is blue], jump five times⟧
        = 1 iff you must jump five times if the square is blue$_\exists$ and no other colour.

In fact, Exh is generally obligatory and necessarily local with summative predicates, including in non-DE environments—and we even saw that a global Exh must be ruled out even in some monoclausal sentences.

    But it cannot be the case that Exh is obligatory and necessarily local with summative predicates under *not*: the right meaning for colour terms in negative sentences is obtained with no Exh whatsoever (32a), and cannot be obtained with an Exh below *not* (32b) (or—not shown here—with an Exh above *not*).

(32)   a. ⟦not [the square is blue]⟧ = 1 iff ¬blue$_\exists$(s).

        b. ⟦not [Exh$_{\mathrm{ALT}}$ [the square is blue]]⟧
           = 1 iff ¬[blue$_\exists$(s) ∧ ¬white$_\exists$(s) ∧ ¬red$_\exists$(s) ∧ ¬…].

Negation looks like an exception to the generalization that Exh is obligatorily present and local with summative predicates.[3]

This discrepancy between negative and non-negative DE environments can be captured by replacing Exh with Pexh. Since Pexh does not affect falsity conditions and *not* reverses truth and falsity conditions, having Pexh below negation does not influence the global truth conditions:

(33) $[\![$not $[$Pexh$_{\text{ALT}}$ [the square is blue]$]]\!]$
$$= \begin{cases} 1, \text{ iff } \neg\text{blue}_\exists(s); \\ 0, \text{ iff } \text{blue}_\exists(s) \wedge \neg\text{red}_\exists(s) \wedge \neg\text{white}_\exists(s) \wedge \neg \ldots; \\ \#, \text{ otherwise} \end{cases}$$

Further, while Pexh does not affect the truth conditions of negative sentences, it does affect the falsity conditions. Its effect is that the sentence is neither true nor false if the square is partly blue. This is in line with the description by Löbner (2000), and makes non-minimality possible through Križ's mechanism—both welcome results.

In short, with Pexh, the locality constraint on exhaustification can be claimed to be absolute; there is no need to model negation as an exception.

3.4   Section conclusion: new arguments in favour of Pexh

This section has discussed three problems for the Exclusion theory of summative predicates based on a bivalent Exh. They can all be solved by claiming that Exh is trivalent, as well as adopting Križ's theory of non-maximality. These arguments in favour of Pexh are different from those given by Bassi et al. (2021), and should be taken as adding to these authors' argumentation. Solving Problems 1–2 involved using Pexh to create truth-value gaps where truth-value gaps have been described as being intuited; while Bassi et al. (2021) end up creating truth-value gaps for the data they discuss, that is not how their data had previously been described. For these authors, positing truth-value gaps is a means to an end rather than an empirical goal. As for Problem 3, this involved using Pexh to derive a contrast between negation and other DE environments that has not previously been described in the literature on exhaustification; this too is a new kind of argument in favour of Pexh.

This constitutes the main contribution of this paper; we now turn to some consequences of this discussion for homogeneity and non-maximality for plural predication. In particular, by assuming Pexh to be locally present with all summative predications including negative ones (where it is located below *not*) and taking

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3 I tentatively suggest in chapters 2 and 6 of Paillé 2022a that it is not just under *not* that no local Exh is observed in the truth conditions of summative predicates, but more generally with negative-flavoured DE environments (under *not*, *no*, and maybe *doubt* and similar predicates). What matters for summative predicates is whether the environment they are in is positive or negative.

non-maximality to emerge from truth-value gaps (via Križ's mechanism), I have taken non-maximality and non-minimality to be exactly the same phenomenon. This is in contrast to Bar-Lev's (2021) discussion.

4   Local Pexh and plural predication

In many ways, Bar-Lev's account of plural predication via the Inclusion of domain-alternatives (see section 2.2) can co-exist with the Exclusion account of summative predicates, as different analyses of different phenomena. But some of the assumptions in this paper affect Bar-Lev's proposal. In this section, I start by comparing the predictions of Bar-Lev's bivalent exhaustification framework and of what his Inclusion theory would look like with local Pexh and Križ's account of non-maximality (section 4.1). The main difference concerns how non-maximality and non-minimality are derived; in Bar-Lev's bivalent framework, these have different status, but the assumptions in this paper make it possible to derive non-minimality in the same way as non-maximality. From there, I turn to Bar-Lev's two arguments in favour of a difference between non-maximality and non-maximality: data from acquisition (section 4.2) and adult judgments (section 4.3). My discussion will not be entirely conclusive, but will at least show that theorizing a fundamental difference between non-maximality and non-minimality is not a necessary property of exhaustification accounts of homogeneity.

4.1   Non-minimality in bivalent and trivalent frameworks

Consider again positive and negative plural predication:

(34)   a. The children sang.

b. The children didn't sing.

As discussed in section 2.2, Bar-Lev (2021) suggests that the meaning of plural predication is existential; (34b) involves a negation of this existential meaning, while (34a) involves strengthening. Exh includes subdomain alternatives, informally of the form 'child x sang' or 'the subplurality x of children sang'; when all these alternatives are asserted, the resulting meaning is that every child sang. Bar-Lev suggests to obtain non-maximality through the pruning of alternatives (as discussed in section 3.2.2). Informally, if Exh does not compute alternatives about certain children (or amounts of children) in (34a), the meaning will not be universal. For instance, if there are three children and all alternatives about a single child are pruned, Exh only ends up including alternatives about larger-than-one subpluralities of children:

(35)   {~~a sang~~, ~~b sang~~, ~~c sang~~, a or b sang, a or c sang, b or c sang, a or b or c sang}

Asserting that these non-pruned alternatives are true results in the meaning that at least two of the three children sang.

Some assumptions in this paper affect the Inclusion theory of plural homogeneity. I claimed that exhaustification is trivalent, so there must be truth-value gaps in (at least) positive sentences containing plural predication. I also adopted Križ's mechanism deriving non-maximality from heterogeneity-gaps; this is predicted to kick in with all truth-value gaps, whether they come from summative predicates or plural predication. This means that deriving non-maximality through pruning is no longer necessary, although nothing rules it out as a possibility. But my assumption that affects Bar-Lev's proposal most substantially concerns non-minimality. On Bar-Lev's approach, since Exh is absent from negative sentences, non-minimality cannot be derived through pruning. He suggests that non-minimality arises instead due to non-default covers, which he claims makes non-minimality more difficult to intuit than non-maximality (see his section 5.3).

My claim that negative sentences with summative predicates involve Pexh below *not* does not predict that Pexh would also necessarily occur below *not* with plural predications; (34b) might be optionally non-exhaustified. But we certainly expect the *possibility* of Pexh being present below *not*, as sketched out in (36) (where I assume a discourse with two children):

(36)   a. not [Pexh$_{\text{ALT}}$ [the children sang]]

b. ALT $= \{a$ sang, b sang, a or b sang$\}$

c. $[\![\text{Pexh}_{\text{ALT}} \text{ [the children sang]}]\!] = \begin{cases} 1, \text{iff } a \text{ sang and } b \text{ sang;} \\ 0, \text{iff neither } a \text{ nor } b \text{ sang;} \\ \#, \text{otherwise} \end{cases}$

d. $[\![\text{not [Pexh}_{\text{ALT}} \text{ [the children sang]]}]\!] = \begin{cases} 1, \text{iff neither } a \text{ nor } b \text{ sang;} \\ 0, \text{iff } a \text{ sang and } b \text{ sang;} \\ \#, \text{otherwise} \end{cases}$

As such, I clearly predict that it should be possible for non-minimality to arise from Križ's mechanism just like non-maximality.

On the other hand, no clear prediction is made about whether non-minimality should be more difficult to intuit than non-maximality. Without better understanding the locality of this exhaustification, it is not clear whether (36) is the only possible option or whether it is possible for negative sentences to be non-exhaustified as posited by Bar-Lev (2021). If non-minimality is indeed more difficult to intuit, one could claim this is because inserting Pexh below *not* is not the default parse; if non-minimality is not more difficult to intuit, one could claim this is because inserting Pexh below *not* is obligatory due to the locality constraint (i.e. Exh below *not* is the default parse), with non-minimality therefore arising identically to non-maximality.

We now discuss Bar-Lev's two arguments for a contrast between non-maximality and non-minimality.

4.2 Argument 1: Acquisition data

Summarizing experimental findings by Tieu, Križ & Chemla (2019), Bar-Lev's first argument for an asymmetry between non-maximality and non-minimality comes from judgments by children: "while some children can be categorized as interpreting definite plurals existentially and some as interpreting them universally in positive sentences, no child can be categorized as having a weak negated universal interpretation in negative sentences" (Bar-Lev 2021: 1051). This is a welcome for all theories claiming that homogeneity involves weak lexical meaning that is strengthened in positive sentences: the acquisition data can be thought of as some children simply not computing the exhaustification (Bar-Lev 2021). If exhaustification is bivalent and absent from negative sentences (as in Bar-Lev's framework), children's behaviour is exactly as expected if they sometimes do not compute it: this gives them weak meanings for positive sentences, while necessarily having strong meanings for negative sentences. If, on the other hand, exhaustification is trivalent and (for homogeneity effects) present in negative sentences below negation, children's behaviour makes sense too. On this approach, children rigidly interpret negative sentences as negated existentials because Pexh does not affect the truth conditions of negative sentences. Regardless of whether or not a child computes the exhaustification, the truth conditions of negative sentences are the same.

4.3 Argument 2: Adult judgments about non-minimality

Where the predictions of Bar-Lev's Exh account and the local-Pexh account might diverge (depending about whether Pexh is obligatorily present under *not*) concerns adult judgments about non-minimality.

Non-minimality is about as easy to intuit as non-maximality in many examples, such as the following two (where I follow Bar-Lev (2021) in ensuring that the relevant plurals are interpreted below negation by having a variable bound by a negative quantifier); (37a) is mine and (37b) is given by Bar-Lev (2021: 1080).

(37)   a. i. SCENARIO: *Five spies meet to view a document stored on an encrypted computer. Twenty-five passwords must be entered in sequence to view the file; each spy has five passwords. While every spy has shown up with some of their passwords, none of them showed up with all five of them, so they cannot unlock the computer to view the file.*

   ii. None of the spies showed up with their passwords.

> b. i. SCENARIO: *Each contestant in a pie eating contest is provided with twenty-five pies. In order to move on to the next round, a contestant must eat at least twenty pies; they get more points for eating more.*
>
> ii. No contestant ate their pies.

(37a-ii) is paraphrasable as 'none of the spies showed up with all of their passwords,' and (37b-ii) as 'no contestant ate at least twenty pies' (Bar-Lev 2021: 1080).

However, Bar-Lev provides cases where some speakers intuit non-maximality in a positive sentence (38a), but not non-minimality in its negative counterpart (38b).

(38) SCENARIO: *The kids are required to take at least two of the four vitamins I gave each of them.* (Bar-Lev 2021: 1053)

a. All of the kids took their vitamins.

b. None of the kids took their vitamins.

Bar-Lev (2021: 1053) reports that (38a) is paraphrasable as 'all of the kids took at least two vitamins,' while (38b) means that none of them took any vitamin, rather than none of them meeting the requirement of taking two. This is not a judgment I share, and Bar-Lev (2021: 1079) writes that whether or not (38b) invites a non-minimal reading might hinge on whether one makes "the reasonable assumption that taking one vitamin is better than taking none." Whether a world in which all the children took one vitamin is in the same cell of the QUD as a world in which no child took any vitamins depends on whether taking one vitamin is substantially better than taking none. Thus, to the extent that some speakers find non-minimality difficult to intuit in (38b), this could be because the worlds corresponding to the truth conditions (that no child took any vitamin) are in their own cell. Making it more explicit that only taking one vitamin (or pill in (39)) is not better than taking zero should remove this uncertainty:

(39) a. SCENARIO: *The kids have a disease whose treatment involves taking two different pills. Each pill does nothing by itself; only taking one of the two does nothing to help their treatment.*

b. None of the kids took their pills.

To my ear, non-minimality is straightforwardly intuited in (39b), which means that none of the kids took the two pills needed for effective treatment.

So far, there is no reason to break the symmetry between positive and negative sentences predicted by the local-Pexh account. The last example Bar-Lev gives I would like to discuss is (40):

(40)   a. All of the kids came with their parents to the meeting.

      b. None of the kids came with their parents to the meeting.
        (Bar-Lev 2021: 1053)

Non-maximality is more easily intuited in (40a) than non-minimality in (40b). There are two scenarios to consider: that the kids had to come to the meeting with all their parents, or that the kids had to come to the meeting with one or more parent. The meaning intuited for the positive (40a) involves the kids meeting the requirement, whether that means coming with one parent or two. In contrast, the negative (40b) is less context-dependent, rigidly implying that no child showed up with any parent.

    There are two points to make about this example. First, since the children are children, we might follow Bar-Lev's point about (38) that regardless of the formal requirement, showing up to the meeting with one parent is better than being unaccompanied. Second, it might help to be clear about the (negative) consequence associated with not following the requirement. (41) makes both those changes, and non-minimality is more readily available.

(41)   a. SCENARIO: *17- and 18-year-old high school students must come to a meeting about their upcoming graduation with both their parents. A draconian school administrator has ruled that students who show up with less than both their parents will be barred from graduating.*

      b. None of the kids came with their parents to the meeting.

    While (40) shows that non-minimality is a bit less readily available than non-maximality, the difference is weak and easily overridden by minor contextual manipulations. Given that there is a difference between non-maximality and non-minimality, but it is a small difference, it is not clear to me whether negative plural predications should be analysed as always having Pexh below *not* (predicting a complete parallel between non-maximality and non-minimality), or Pexh below *not* being a non-default option (predicting that non-minimality should be 'difficult').

4.4   Section conclusion

The assumptions in this paper open the possibility that plural predication, even if due to the Inclusion of domain-alternatives (Bar-Lev 2021), might give rise to truth-value gaps in positive and negative sentences, with non-maximality and non-minimality arising from there. Data from acquisition are compatible with this view; since Pexh affects the truth conditions of positive but not negative sentences, a child who does not exhaustify plural predication will have strong truth conditions for negative sentences but weak ones for positive sentences. As for adult language, with the right contextual manipulations, non-minimality does arise quite readily in many

cases, apparently in line with the view that it is the same effect as non-maximality. At the same time, non-minimality is somewhat more difficult to intuit, and some explanation of this is needed; perhaps Pexh below *not* is a non-default parse, or perhaps something else is at play.

5 Conclusion

What is the nature of exhaustification? This paper has aimed to give support to the proposal by Bassi et al. (2021) that it is trivalent rather than bivalent. Empirically, I focused on the Exclusion account of summative predicates first proposed by Harnish (1976) and Levinson (1983). This Exclusion theory faces three difficulties: it does not predict heterogeneity-gaps, non-maximality, or a contrast between negation and other DE environments. I showed that these shortcomings can all be overcome by assuming that Exh is trivalent.

In doing so, I followed Križ (2015) in taking non-maximality to arise from truth-value gaps, with trivalent Exh present even in negative sentences. These assumptions led to the prediction that non-maximality and non-minimality should be equally available with summative predicates, which I assumed to be correct. They also led to the possibility of symmetry between non-maximality and non-minimality in exhaustification-based accounts of plural predication, which might well also be right.

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