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Co-predications and the quantificational force of summative predicates

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Abstract

In the literature on homogeneity, summative predicates have been described as quantifying universally over their argument's parts in positive sentences, while being negated existentials in negative sentences. In this article, I provide a fuller picture of these predicates' quantificational force in positive sentences through various 'co-predications'—sentences in which two summative predicates are predicated of the same individual. In some co-predications, summative predicates are universal; in others, they are weaker, while remaining stronger than existential. In light of this new empirical paradigm, I suggest that summative predicates are lexically existential, but are exhaustified so as to exclude other same-class predicates. In addition to making this proposal, I also show that no other theory of homogeneity can capture the co-predicational paradigm.

1. INTRODUCTION

A topic of much recent interest in semantics has been the observation that predicates composing with pluralities have universal force in positive sentences (1a), but are negated existentials in negative sentences (1b).

(1) a. Adam saw the children.
$\approx$ he saw all of the children
$\not\approx$ he saw at least some of the children
b. Adam did not see the children.
$\not\approx$ he did not see all of the children
$\approx$ he saw none of the children

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This HOMOGENEITY effect is observed not only with pluralities, but also within atoms with so-called SUMMATIVE predicates, such as colour adjectives (Löbner 2000; Spector 2013; Križ 2015). These are predicates like *green* or *wooden* that are true of an individual by virtue of being true of that individual's material parts; they are opposed to INTEGRATIVE predicates (like *flag* or *happy*) that hold of an individual as an undivided whole.

(2) a. The flag is green.
$\approx$ all of the flag is green
$\not\approx$ at least some of the flag is green
b. The flag is not green.
$\not\approx$ not all of the flag is green
$\approx$ none of the flag is green

Call the paradigms in (1) and (2) PLURAL and SUBATOMIC homogeneity effects.

This article makes two general contributions to the study of subatomic homogeneity. The first contribution is to outline a new empirical paradigm expanding empirically on the quantificational force of summative predicates in positive sentences. This new paradigm is a set of constructions I will call CO-PREDICATIONS, in which more than one summative predicate is attributed to the same individual. A puzzle emerges due to the different quantificational force obtained in different co-predications: certain co-predications maintain the universal force of summative predicates even if this creates inconsistency within a single sentence (3a), but in other in co-predications, summative predicates are intuited as weaker and therefore consistent (3b).

(3) TWO KINDS OF CO-PREDICATIONS:
a. *Co-predications where summative predicates are universal and inconsistent:*
(i) #The white flag is green.
(ii) #The white green flag is at half-mast.[1]
b. *Co-predications where summative predicates are non-universal and consistent:*
(i) The white flag is also green.
(ii) The flag is white and green.

While the literature on homogeneity has discussed the existence of weak quantificational force in positive sentences as arising from particular discourse contexts (so-called NON-MAXIMALITY, discussed in section 2), (3b) shows weak quantificational force of a discourse-independent nature. This is not something that the literature has attempted to capture in positive sentences. Clearly, the fact that this weak meaning is not always observed in co-predications (3a) makes this explanandum all the more challenging.

The second contribution of this article is more theoretical. I will both suggest a new way to understand subatomic homogeneity, building on work by Harnish (1976) and Levinson (1983), and use the new co-predicational paradigm to evaluate various other theories of homogeneity. I will show that only the modified Harnish–Levinson proposal can account for co-predications. These authors suggest that summative predicates are lexically existential:

(4) $[\![\text{green}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{green}(y)] \equiv \lambda x.\mathsf{green}_{\exists}(x)$.

1 Some languages (such as German or Polish) allow *compound* colour terms, but not *stacked* ones like in (3a-ii) (Paillé 2022b:§6.3.4).

This obtains the truth conditions of negative sentences immediately. For positive sentences, Harnish and Levinson suggest that colour terms are strengthened through the exclusion of other colour terms (5); if the flag is partly green and has no other colour, it must be all green. I will refer to this as the Exclusion theory of subatomic homogeneity, and model exclusion through the grammatical Exh operator of Chierchia *et al.* (2012).

(5) a. Exh_{ALT} [The flag is green].
b. ALT = {The flag is green$_\exists$, The flag is white$_\exists$, The flag is red$_\exists$, ...}
c. $[\![(5a)]\!] = 1$ iff $\textsf{green}_\exists(f) \wedge \neg\textsf{white}_\exists(f) \wedge \neg\textsf{red}_\exists(f) \wedge \ldots$

I will modify the Harnish–Levinson account in two substantial ways. First, the Exclusion account can only work with a stipulation that the exhaustification is necessarily computed locally to the colour term, something I will model through an Agree relation between colour terms and Exh. Second, I will use the trivalent exhaustivity operator ('Pexh') of Bassi *et al.* (2021) in order to capture HETEROGENEITY-GAPS, the truth-value gaps that arise when an individual is non-homogeneous vis-à-vis a summative predicate. Following this endorsement of this Exclusion theory, I will spend the remainder of the article showing that no other currently existing theory of homogeneity can capture all the co-predications in (3).

This article is organized as follows. Section 2 discusses the classic homogeneity paradigm and non-maximality with summative predicates. Section 3 then expands on this with the co-predication paradigm—the central empirical contribution of this article. From there, I turn to three sets of theories in sections 4–6. The Exclusion theory just described is taken up in section 4. Then, section 5 discusses two theories deriving universal quantification in positive sentences semantically (e.g. Löbner 2000, Bar-Lev 2021); both create more contradictions among co-predications than observed. Section 6 discusses theories involving semantic underspecification (e.g. Krifka 1996, Križ and Spector 2021); they predict too few contradictions. Finally, section 7 concludes with comments on what unites plural and subatomic homogeneity, and on additional benefits of the Exclusion theory beyond quantification in summative predicates.

2. BACKGROUND ON SUMMATIVE PREDICATES' QUANTIFICATION

We have already observed the homogeneity paradigm with summative predicates; (6) is repeated from (2). This section expands empirically on the paradigm in (6) in two ways, both of which are well established in the literature. Section 3 will then add new observations to this discussion.

(6) THE CLASSIC HOMOGENEITY PARADIGM:
a. The flag is green.
$\approx$ all of the flag is green
$\not\approx$ at least some of the flag is green
b. The flag isn't green.
$\not\approx$ not all of the flag is green
$\approx$ none of the flag is green

2.1 *Heterogeneity-gaps*

The first way that (6) oversimplifies the empirical picture is that it only describes the TRUTH CONDITIONS of the sentences. Truth conditions will be the main focus of this article, but a theory of homogeneity would be incomplete without capturing that these sentences' meanings are more complicated than that; in particular, their truth and falsity conditions are not complementary. One way to appreciate this comes from Löbner's (2000) suggestion that the falsity conditions of a sentence are the same as the truth conditions of its negation; from there, it follows that (6a) has the falsity conditions that the flag is not green at all, and (6b) has the falsity conditions that the flag is entirely green. Thus, if the flag is partly but not fully green (what I will refer to as 'non-homogeneous cases'), the sentences in (6) will be neither true nor false (see Križ 2015 for a recent defense of this, and Bar-Lev 2021 for an alternative view). I will refer to these sentences as having HETEROGENEITY-GAPS—truth-value gaps in non-homogeneous cases.

2.2 *Non-maximality*

The second way that (6) by itself is an oversimplification is that it does not show how malleable summative predicates' quantificational force can be. To see this, first consider a colour adjective being predicated of something other than a flag; the empirical picture can become considerably more complex (see Kennedy and McNally 2010 and citations therein). Indeed, colour terms are often used to refer only to some salient component of their subject; this is something that cannot be observed with examples involving flags, because flags lack components other than those created by the colours themselves. For example, the two sentences in (7) could both be true of the same grapefruit, if its skin is yellow (7a) and its flesh is pink (7b).

(7) a. The grapefruit is yellow.
b. The grapefruit is pink.

Thus, summative predicates can be less than universal even in positive sentences. I write 'less than universal' because the colour terms in(7) are still stronger than existential; (7a) means the skin is entirely yellow, while (7b) means the flesh is entirely pink.

The observation of less-than-universal quantificational force in positive sentences does not only arise based on the nature of the argument (whether it is a flag or a grapefruit, in the above examples), but also on discourse context. Out of the blue, (8) is most readily understood as involving universal quantification by *red*, but this can change with some context (9).

(8) Your shirt is red.

(9) a. SCENARIO: *We are entering a bullfighting arena. Visitors are not permitted to wear any red, but my shirt is half red, half white. A security guard says:*
b. Your shirt is red, you cannot enter the arena.

In this example, it happens to be that, unlike the examples in (7) (which both require a certain part of the grapefruit—its skin or flesh—to be entirely yellow/pink), (9) is outright existential. Either way, in both cases, the meaning is not universal.

The observation of less-than-universal quantification in positive sentences with predicates displaying homogeneity is called NON-MAXIMALITY. This phenomenon has received considerable attention in the literature on plural homogeneity (e.g. Dowty 1987, Lasersohn 1999, Brisson 1998, 2003, Malamud 2012, Schwarz 2013, Križ 2015, Križ and Spector 2021, Bar-Lev 2021). Križ (2015) points out that (10) could be true and felicitous in many discourse contexts even if only most (e.g. eight out of ten) of the professors smiled.

(10) The professors smiled.

In addition to observing non-maximality in positive sentences, we also find NON-MINIMALITY in negative ones:

(11) a. SCENARIO: *Tennis courts in a rich neighbourhood only allow people in if they are dressed exclusively in white. You try to go to a tennis shirt with a half white, half red shirt. A security guard says:*
b. You can't go in, your shirt isn't white.

Here, *not white* means 'not entirely white' rather than 'not white at all'. Presumably, this non-minimality, being conditioned by discourse context, is the mirror image of the non-maximality observed in (9) (but see Bar-Lev 2021). To keep things simple, I will focus on non-maximality in positive sentences (see Paillé 2023b on non-minimality with summative predicates).

From the observation that non-maximality can arise due to discourse factors (9), I assume that the same holds for the examples with grapefruits (7), even if no discourse context was necessary to observe it there. The idea is that one can utter (7a) if what matters for the conversation is the colour of the exterior of the grapefruit, while one can utter (7b) if what matters is the edible interior. Hence, even though observing non-maximality requires explicit discourse context in some cases but not others, we can safely assume non-maximality is generally conditioned by discourse. The flip-side is that, since this article will not generally focus on non-maximality (qua discourse-conditioned phenomenon), we can control for it by using examples with objects like flags or shirts that, without particular discourse contexts, resist non-maximality.

What causes non-maximality to arise in discourse? Prima facie, the most obvious way to think about it would probably be as domain-restriction. However, Križ (2015) argues against such an analysis for plural homogeneity, and his arguments extend to summative predicates. One of his arguments comes from discourse anaphora. Building on the discourse in (10), which is felicitous even if not all the professors smiled, we can go on to refer to all of the professors with *they*:

(12) The professors smiled. Then they (all) stood up and left the room. (Križ 2015:75)

The same goes for summative predicates. Consider (13), where *red* is not universal vis-à-vis the entire car (for instance, the tyres and steering wheel are presumably not red) but only its painted exterior.

(13) The car is red.

Much like with plural predication (12), the entire car can still be referred to by *it* in a discourse continuation (14). The crucial property of (14) is that the predicates in the second sentence are all summative, and they can be true of all the parts of the car (including the steering wheel, for instance) even while *red* in the first sentence is not.

(14) The car in this movie is red. It is (entirely) {CGI, animated, cheap plastic, ...}.

Likewise, exceptives can be used to make explicit what the speaker has in mind in their non-maximal use of a summative predicate (15a), while exceptives result in a non-sequitur in real cases of domain-restriction (15b) (Križ 2015).

(15) a. The car is red, except for the grey roof.

b. (i) Scenario: *We are at McGill University and discussing how the students at this university reacted to a new announcement by the administration.*
(ii) The students are happy, #except those at the Université de Montréal.

Rather than taking non-maximality to be a case of domain-restriction, Križ (2015) suggests that non-maximality arises from the existence of heterogeneity-gaps. The connection between non-maximality and heterogeneity-gaps can be appreciated from the fact that both of these phenomena disappear with *all* (Križ 2015):

(16) a. All the professors smiled. (false if eight out of ten smiled)
b. All of the shirt is red. (false in the bull-fighting scenario (9a))

The basic insight of Križ's (2015:76ff) theory is that sentences that are neither true nor false in the world of utterance w_0 can be used felicitously if w_0 is, for the purposes of the conversation, equivalent to a world in which the sentence was true. The first ingredient of this theory is the standard assumption (e.g. van Rooij 2003) that a question under discussion (QUD) partitions worlds by how they resolve it. Let us consider the sentence (17) with a toy model of three worlds (18) corresponding to different amounts of red on the shirt:

(17) The shirt is red.

(18) $$\begin{Bmatrix} w_1 : & \text{the shirt is all red,} \\ w_2 : & \text{the shirt is half red,} \\ w_3 : & \text{the shirt is not red at all} \end{Bmatrix}$$

If the QUD is 'How much red does the shirt have?' or 'What does the shirt look like?', all of these worlds are in their own cell—they all correspond to different answers to the QUD. On the other hand, if the QUD is 'Does the shirt have any red on it?', w_1 and w_2 both correspond to the answer 'yes', so they are in the same cell.

From here, Križ (2015) suggests to take to the letter Grice's (1975:75) maxim of Quality, which Grice phrased as 'Do not say what you believe to be false' rather than 'Say what you believe to be true'—these are not equivalent in a trivalent semantics. Križ (2015) suggests that speakers can utter sentences that are neither true nor false in the world of utterance, as long as they are true in some of the worlds in the cell of the partition containing the world of utterance. That is, a sentence must correctly identify the cell containing the real world, but does not need to identify the real world as such. Thus, QUD permitting, speakers may say things that are neither true nor false in the real world.

2.3 *Section summary*

This section has elaborated on the basic homogeneity paradigm (6) with two observations taken from the homogeneity literature. These observations show that it is an oversimplification to describe summative predicates simply as universal in positive sentences and negated existentials in negative ones. First, they display heterogeneity-gaps (their truth and falsity conditions are not complementary), and second, discourse factors can affect their quantificational force. The rest of this article will control for discourse factors by using sentences with flags as their subjects, which do not readily lend themselves to non-maximality out of the blue.

Even with these complications, if we take for granted Križ's (2015) theory of non-maximality, it remains that on the standard view of summative predicates, all that one needs to explain about their quantificational force is the following: in positive sentences, summative predicates have universal truth conditions and negated-existential falsity conditions, with a heterogeneity-gap for non-homogeneous cases; and the reverse holds for negative sentences. But we now turn to new data showing that the empirical picture is in fact much more complicated.

3. TWO TYPES OF CO-PREDICATIONS

This section expands empirically on the quantificational force of summative predicates in positive sentences. We will see that, when two summative predicates from the same class (e.g. colour terms) are predicated of the same individual, or are CO-PREDICATED, their observed quantificational force depends on the manner in which they are co-predicated. Indeed, in some co-predications, summative predicates are universal just as in the basic homogeneity paradigm; as such, they are inconsistent with one another, and a contradiction emerges. In other cases, namely with *and* or *also*, co-predication is consistent, because the summative predicates are intuited as EXISTENTIAL-PLUS: weaker than universal, but stronger than merely existential (together, they entail that all parts of the individual are covered by one or the other predicate). We take these two classes of co-predications in turn.[2]

3.1 *When co-predicated summative predicates are inconsistent*

In some co-predications, both summative predicates are intuited as universal, and a contradiction arises. Empirically, there are two such cases: when one summative predicate is attributive while the other is predicative (19a), and when two summative predicates are stacked on a single noun (19b).

(19) a. #The white flag is green.
b. #The green white flag is high.

In both cases, a sentence-internal contradiction is intuited due to the meanings clashing with world knowledge; we do not conceptualize of surfaces as possibly being entirely of two different colours.

While it is clearly in line with intuitions that the data in (19) are semantically deviant due to the quantificational force of the summative predicates, a sceptic could try to explain the deviance as arising from some notion of asymmetry between the predicates. Perhaps the problem for (19a) is that it is odd to place one colour term in the subject and the other in the predicative position, and the problem for (19b) is that *green* and *white* are at different syntactic heights. However, this cannot be maintained as a sufficient explanation. For (19a), the presence of one colour term in the subject and another in the *v*P cannot be the cause of the deviance, because there are other sentences with similarly 'distributed' colour terms that are acceptable; this will be shown in section 3.2. As for (19b), certain non-summative predicates from a given conceptual class (much like *white* and *green* come from the conceptual class of colour terms) can in fact be stacked in this way without a contradiction, so it is not the stacking as such that should be blamed:

2 I focus exclusively on mono-clausal co-predications in this article; nothing hinges on this.

(20) The plates on the left are warm, and those on the right are cold. Among the warm ones, there is a range of temperatures from mild to scorching hot. Would you like to eat with a medium warm plate, or **a hot warm plate**?

The deviance of stacked colour terms is maintained even in discourses similar to (20):

(21) The plates on the left are at least partly green, and the ones on the right have no green at all. Among the partly green plates, you can choose which other colour is present on the plate. #Would you like to eat with **a white green plate, or a black green plate**?

Thus, the semantic deviance of the examples in (19) is due to the quantificational force of the summative predicates, rather than some accidental property of these sentences. The important conclusion is that co-predicated summative predicates are at least sometimes intuited as universal, and therefore inconsistent.

3.2 *When co-predicated summative predicates are consistent*

There are also at least two cases where co-predication is consistent, with the summative predicates interpreted as non-universal and no contradiction arising. This occurs when co-predications are mediated by additive particles or *and*; we take these in turn.

3.2.1 *Additive particles* I just discussed deviant examples like (22) as an example of inconsistent co-predication:

(22) #The white flag is green.

What is striking is that an additive particle like *also* can remove the inconsistency of (22). In (23), the additive is intended to refer anaphorically (Kripke 2009) to the clause-internal antecedent *white*, rather than to any material from the preceding discourse.

(23) The white flag is also green.

The meaning of (23) is that the flag is partly white and partly green, and has no other colour. As such, while the quantificational force of the colour terms is not universal, it is more than just existential. Call it EXISTENTIAL-PLUS; another term would be ‘exclusive existential’.

Examples like (23) may require a bit of work to explain why one predicate is in the subject position and the other is in the *v*P. Some speakers may find the example easier with an indefinite:

(24) Some white flags are #(also) green.

Either way, it is not difficult to come by scenarios that make it natural to put one colour term in the subject and the other in the *v*P. For example:

(25) a. SCENARIO: *We are at a plant that specializes in recycling cloth; pieces of cloth must be sorted by colour. There is a pile of flags, most of which are entirely white, but a few of which are both white and green. The boss tells a worker that they need to remove all the green parts from the otherwise white flags:*
b. Some of the white flags are #(also) green, so I want you to cut off the green parts.

An important empirical puzzle touching on (23), about which I will have nothing to say, is why not all summative predicates behave in the same way. While colour terms can be intuited as existential-plus when co-predicated via *also* (26a), this is not the case with material terms (26b). This is in spite of both colour terms and material terms otherwise patterning similarly (they both display classical homogeneity and pattern together for all other types of co-predications).[3]

(26) a. The white flag is also green.
b. #The metal table is also wood.

It is not clear why the two classes of summative predicates should pattern differently. I leave this for future work, and will focus on colour terms in this article. But whatever the nature of the class of summative predicates that can compose consistently via *also*, it includes more than just colour terms. The summative predicates *live-action* and *animated* are one example:

(27) Some live-action movies are #(also) animated.

Without the additive, (27) means that the film is both (just about) entirely live-action throughout and entirely animated throughout—an impossible state of affairs, of course.

Many speakers report that sentences with colour adjectives joined by *also* need the colour term in the subject to cover a larger part of the flag than the colour term in the *v*P. That is, the felicity/truth conditions of (28) (for these speakers) are not quite identical, as suggested in the right-hand parentheses.

(28) a. The white flag is also green. (SALIENT READING: the flag is mostly white)
b. The green flag is also white. (SALIENT READING: the flag is mostly green)

But in reality, greater coverage of parts is only one of many reasons why a speaker could choose to put one colour term in the subject, and the other in the *v*P. Consider (29), for example, which motivates this 'distributed' use of colour adjectives without implying that the subject-internal colour term covers more of the flag:

(29) a. SCENARIO: *There are two flags in front of Adam and Jade; Flag 1 is entirely orange, while Flag 2 is 20% white, 80% green. From his position, Adam cannot see that Flag 2 has any green on it.*
b. A.: The white flag is higher than the orange one!
J.: It is! By the way, the white flag is also green.

Similar asymmetries hold with integrative predicates too (see section 7 and Paillé 2022b on *also* with integrative predicates):

(30) a. A futon is a couch that is also a bed.
b. #A futon is a bed that is also a couch.

3 In fact, even the sentence in (26a) is not fully accepted by everyone; some speakers find it more immediately readable as meaning that the fully white flag is somehow 'also' fully green. Still, I have yet to find anyone who fully rejects the sentence or denies that it is significantly better than the counterpart without *also*.

Since it does not touch directly on the quantificational force of summative predicates, I will put aside this asymmetry for the remainder of this article.

3.2.2 *Conjunction* The second kind of co-predication that leads to a consistent, existential-plus meaning for summative predicates is conjunction, as in (31).

(31) The flag is green and white.

The description that (31) is intuited with the colours not covering the entire flag is non-controversial; but what I claim is specifically that this is due to the colour terms themselves being interpreted as weaker than universal, as shown informally in (32a). This is in contrast to a competing analysis by Krifka (1990), sketched out in (32b), where the colour terms are universal and quantification introduced by a non-Boolean *and* is the cause of the consistency in (31).

(32) *How the consistent meaning in (31) arises compositionally*

a. **My claim:**
$\text{white}_{\exists+}$ $\text{and}_{\text{Boolean}}$ $\text{green}_{\exists+}$

b. **Krifka's (1990) claim:**
$\text{white}_{\forall}$ $\text{and}_{\text{non-Boolean}}$ $\text{green}_{\forall}$

We spend the rest of this subsection defending the view in (32a) against (32b).

Krifka's claim is due to an apparent parallel between (31) and conjunction with plural subjects. With plurals, conjoined predicates can be interpreted either as modifying all parts of their plural argument (33a) or as existential-plus (33b), each predicate P and Q being true of some proper part such that $P(x)$ or $Q(x)$ is true of each atomic part x. This latter interpretation can be brought out most clearly by conjoining predicates that cannot both hold of a single individual (at a given time).

(33) a. The planets are big and rocky.
(MOST SALIENT INTERPRETATION: *all the planets are big and all the planets are rocky*)

b. The planets are 3 billion and 5 billion years old. (cf. Krifka 1990)
(ONLY NON-CONTRADICTORY INTERPRETATION: *some of the planets are 3 billion years old, and the rest are 5 billion years old*)

Krifka (1990) suggests to understand the existential-plus meaning in (33b) not as arising from the predicates not distributing universally, but as being due to *and* dividing the subject *the planets* in two parts, with the predicates then distributing universally over those subpluralities. For our purposes, we can simply claim (incorrectly; see e.g. Schmitt 2021) that there are two lexical meanings for *and*, one of which is Boolean and one of which is not:[4]

(34) a. $[\![\text{and}_1]\!] = \lambda P.\lambda Q.\lambda x.\ P(x) \wedge Q(x).$

b. $[\![\text{and}_2]\!] = \lambda P.\lambda Q.\lambda x.\ \exists x', x''[x = x' \oplus x'' \wedge P(x') \wedge Q(x'')].$ (Krifka 1990)

4 For work claiming that there is a single *and* that is underlyingly Boolean, see e.g. Winter 2001, Champollion 2016, and Schein 2017; for work claiming that there is a single *and* that is underlyingly non-Boolean, see e.g. Krifka 1990, Heycock and Zamparelli 2005, and Schmitt 2013, 2019.

On this view (adopted for simplicity), *and* is lexically ambiguous and speakers choose with which *and* to conjoin predicates based on some notion of naturalness or contradiction-avoidance when two predicates are mutually exclusive.[5]

Since, when a conjunction has a plural subject, *and* can break up the subject into two parts (34b), Krifka (1990) suggests that (35) is essentially the same phenomenon. The summative predicates are universal, but and_2 (34b) breaks up the flag into two parts.

(35) The flag is green and white.

Thus, (35) is consistent due to quantification brought in by and_2, not due to the colour terms being weaker than universal:

(36) $\exists x, x'[\iota y[\mathsf{flag}(y)] = x \oplus x' \wedge \mathsf{green}_{\forall}(x) \wedge \mathsf{white}_{\forall}(x')]$ (Krifka 1990:165)

I now turn to showing that this is not the right analysis; in fact, there is no non-Boolean *and* available with atomic subjects like in (35). To see this, let us begin by manipulating the conjunction itself. There are morphologically overt ways of forcing conjunction to be Boolean. At least when it composes with conjoined predicates, *both* forces a Boolean interpretation of a conjunction; this is also the case with *as well as* (see e.g. Schwarzschild 1996:149, Szabolcsi 2015:199, Schmitt 2021:24; but Schmitt (2013:138–139) argues otherwise). (37a) shows that these 'marked' conjunctions are acceptable with a Boolean interpretation, while (37b) shows that they rule out a non-Boolean interpretation.

(37) a. 'BOTH' AND 'AS WELL AS' WITH BOOLEAN CONJUNCTIONS:
(i) The planets are both big and rocky.
(ii) The planets are big as well as rocky.
b. 'BOTH' AND 'AS WELL AS' WITH NON-BOOLEAN CONJUNCTIONS:
(i) The planets are (#both) 3 billion and 5 billion years old.
(ii) #The planets are 3 billion as well as 5 billion years old.

As such, if colour conjunctions were necessarily non-Boolean, *both* and *as well as* should be incompatible with them. This is not the case:

(38) a. The flag is both green and white.
b. The flag is green as well as white.

(38) suggests that the intuited meaning of (35) is actually compatible with a Boolean interpretation of *and*.[6]

5 See Poortman 2017 for discussion of how the choice of predicates in a conjunction affects the likelihood of speakers preferring to interpret a conjunction intersectively.

6 The claim that *both* conjunctions are necessarily Boolean can be complicated by some pragmatic factors. Consider (i), a datapoint whose judgment is due to the world knowledge that no species both barks and crows.

(i) #The animals both barked and crowed.

In fact, *both* can be made acceptable with some contextual work, such as:

(ii) I hate it when the animals keep me awake. Last night, they both barked and crowed!

This is enough to conclude that conjunction allows the co-predication of colour terms in a way that lets them have an existential-plus, rather than universal, meaning. For good measure, let us take the argument one step further. This time, we will manipulate the conjuncts rather than the conjunction, and arrive at the conclusion that colour conjunctions with atomic subjects are only ever interpreted as Boolean. We will conduct a simple test based on the following intuition: if the colour terms in conjunction data were possibly universal, we should be able to modify them to bring out their universal force explicitly without this causing inconsistency. For example, we could modify them with *completely*.

I assume that *completely* asserts that a predicate is true of all parts of the subject (39); the contribution of *completely* may be semantically vacuous, if the predicate is already universal.

(39) $[\![\text{completely}]\!] = \lambda P.\lambda x.\forall y[y \sqsubseteq x \rightarrow P(y)]$.

Therefore, if colour terms are already universal, modifying them with *completely* makes no semantic difference, and the consistent meaning in (40) is predicted with *and*$_2$:

(40) $[\![\text{completely green}_\forall \text{ and}_2 \text{ completely white}_\forall]\!] = \lambda x.\exists x', x''[x = x' \oplus x'' \wedge [\![\text{completely green}]\!](x') \wedge [\![\text{completely white}]\!](x'')]$.

But this prediction does not carry through; making the conjuncts explicitly universal in a colour conjunction (with an atomic subject) leads to a sharp contradiction:

(41) #The flag is completely green and completely white.

(41) can only mean that the entire flag is simultaneously of both colours, contrary to the expectation if a non-Boolean *and* was available (40).

One way to try and avoid (41) as a falsification of Krifka's theory is to claim that (39) is incorrect. Rather, one could hypothesize that *completely P* not only means that *P* is true of all parts of some argument *x*, but also that *P* is true of all the parts of the maximal individual (plural or atomic) that *x* is a part of. A contradiction would obtain in (41) even if *and* was non-Boolean because each conjunct would end up predicating a colour term of all parts of the entire flag. However, this alternative view does not hold up due to (42), where we would now expect a contradiction too: *completely green* and *completely white* would mean that all parts of the entire plural individual are green/white.

(42) The flags are completely green and completely white.
$\approx$ some of the flags are completely green, and the rest are completely white

Thus, (39) is the right definition for *completely*, and it was correct to conclude from (41) that a non-Boolean *and* is not available there, and more generally that it is not possible for a non-Boolean *and* to refer to parts of atomic individuals.

But this is not a counter-argument to the claim that *both* marks conjunctions as Boolean, since the second sentence in (ii) is non-maximal. More specifically, each conjunct is non-maximal and is interpreted existentially. Indeed, the context in (ii) makes it clear that any minimal amount of barking or crowing 'matters' to the speaker because it contributes to keeping them awake. Thus, the meaning is that at least some animals barked, and at least some animals crowed. Since it is possible for the animals to be in the intersection of the sets of things that have at least one barker, and the set of things that have at least one crower, the conjunction can be Boolean.

There is another way one could problematize (41) as an argument against the possibility of a non-Boolean *and* with atomic subjects: an apparently similar test does not hold with plurals (43). Recall that with plurals, I am not arguing against the possibility of non-Boolean interpretations of conjunctions. In (43), each conjunct contains a token of *each* instead of *completely* because *each* quantifies universally over individuals rather than subatomic parts.

(43) #The flags are each white and each green.

(43) might be expected to be consistent given the availability of a non-Boolean *and* with plural subjects: it would mean that each atomic flag of one subset of the flags is fully green, while each atomic flag of another subset of the flags is fully white (44). In (44), subscript AT stands for 'atomic'—'$Part_{AT}(x)$' is the set of atomic parts of x, and '$\sqsubseteq_{AT}$' is the 'atomic-part-of' relation.

(44) a. $[\![\text{each } P]\!] = \lambda x : |Part_{AT}(x)| > 1.\forall y[y \sqsubseteq_{AT} x \rightarrow [\![P]\!](x)]$.
b. $[\![\text{each white and}_2 \text{ each green}]\!]$
$= \lambda x.\exists x', x''[x = x' \oplus x'' \wedge [\![\text{each white}]\!](x') \wedge [\![\text{each green}]\!](x'')]$.

If so, there is either something wrong with my test, or with my suggestion that a non-Boolean *and* exists with plural but not atomic subjects.

However, floating *each* has independently been argued to necessarily take as its first argument the DP subject, its 'associate'. (44a) and (44b) are incorrect. Or rather, on the proposal by Doetjes (1997) and Fitzpatrick (2006), floating *each* takes a *pro* DP co-indexed with the associate. This view is motivated by a restriction against A′-movement of the associate, agreement with the associate on the floating quantifier in languages like French, and overt clitic doubling of the associate on the floating quantifier in languages like Hebrew. Concretely, Fitzpatrick (2006:81) gives (45a) the LF in (45b) (and assumes the same syntax for *each*).

(45) a. The students will have all had lunch.
b. [$_{DP}$ The students] λ_1 will have [$_{vP}$ [all *pro*$_1$] λ_2 [$_{vP}$ t_2 had lunch]].

In (43), then, each token of *each* has as its first argument a *pro*, each of which is co-indexed with the associate *the flags*. This means that each conjunct carries the entailment that it is every individual flag that is green/white. We therefore expect a contradiction, even if a non-Boolean *and* is available with plurals. Finally, all of this is in contrast to the data with *completely* (41), since *completely* takes as its first argument a predicate rather than a *pro* co-indexed with the predicate's argument (39).

In sum, not only is a Boolean interpretation available with colour conjunctions, it is in fact the only possibility. Thus, what we are observing in the co-predication in (46) is a pair of summative predicates that are non-universal and conjoined intersectively.

(46) The flag is white and green.

The flag is in the set of partly white things and the set of partly green things, and furthermore, it has no other colours.

Of course, there is an important question about *why* a non-Boolean *and* is unavailable for examples like (46). There is a literature on conjunction and whether it is underlyingly Boolean or non-Boolean; some authors (e.g. Schmitt 2021) claim that it is underlyingly non-Boolean. Notably, most of these works focus on conjoined predicates with *plural* subjects. It could be that *and* is only sensitive to the atomic parts of plurals, and not the subatomic parts of atoms; even if it is underlyingly non-Boolean with plurals, this could cause it to act as if it was Boolean with atoms. Either way, what matters for our purposes is that, descriptively, *and* is Boolean in (46) and, therefore, we know we are observing non-universal summative predicates.[7]

3.3 *Section summary: the co-predication paradigm*

In this section, I elaborated on the quantificational force of summative predicates in positive sentences. Beyond the initial simple observation that summative predicates are universal in sentences like (47), their quantificational force shows surprising behaviour in co-predications.

(47) The flag is green.

In particular, co-predicated colour terms sometimes maintain the universal force observed in (47), giving rise to sentence-internal contradictions (48a). But other co-predications result in colour terms' quantificational force being weaker than in (47); it is existential-plus rather than universal (48b).

(48) Two kinds of co-predications:
   a. *Co-predications where summative predicates are universal and inconsistent:*
      (i) #The white flag is green.
      (ii) #The white green flag is at half-mast.
   b. *Co-predications where summative predicates are existential-plus and consistent:*
      (i) The white flag is also green.
      (ii) The flag is white and green.

All of this places a new empirical burden on theories of homogeneity. It is not enough to describe the quantificational force of summative predicates as being universal in positive sentences and a negated existential in negative sentences (49) together with heterogeneity-gaps rather than falsity for non-homogeneous cases.

7 A reviewer asks about examples like (i) and whether they counter my generalization that *and* is necessarily Boolean with atomic subjects:

(i) The designers take the dress to be green and white.

The reviewer suggests that (i) could mean that some designers take the dress to be entirely green, and other designers take it to be entirely white—and that this is due to a non-Boolean *and*, despite *the dress* being atomic. If analysing this as a non-Boolean *and* is the right analysis (rather than e.g. analysing (i) as conjoined non-maximal clauses: 'some designers take it to be green, and some to be white'), the claim in the main text would need to be modified so that the presence of a plural upstairs (*the designers*) makes it possible to have a non-Boolean *and* in this case. While this would add complexity to the empirical picture, it would not falsify the claim that *and* in sentences like (ii) with no upstairs plural can only be Boolean.

(ii) The flag is green and white.

(49) a. The flag is green.
$\approx$ all parts of the flag are green
b. The flag is not green.
$\approx$ no part of the flag is green

In fact, summative predicates can also be existential-plus rather than universal in positive sentences.

There is a third type of co-predication that I do not discuss in this article, namely disjunctions. These are consistent, while having universal colour terms.

(50) The flag is white or green.
$\approx$ the flag is either only white, or only green

Naturally, disjunctions' consistent meaning is due to *or* rather than the quantificational force of the colour terms—hence why I do not focus on these.

We now turn to three sets of theories of homogeneity, with a focus on how and whether they can handle co-predications. I start with the theory I will defend, namely the Exclusion account (section 4), before turning to alternatives in sections 5 and 6. Other than the Exclusion theory, all currently existing theories fail to capture all the co-predications; the ones discussed in section 5 predict co-predications to be contradictory across-the-board, while those discussed in section 6 predict them to be consistent across-the-board.

4. THE EXCLUSION THEORY

In this section, I build a theory of subatomic homogeneity that can capture the co-predication paradigm. I start by overviewing how Harnish (1976) and Levinson (1983) suggest to capture colour terms' universal meaning in positive sentences; they accomplish this through the exclusion of other colour terms (section 4.1). I immediately reframe their discussion in the terms of the grammatical theory of exhaustivity (Chierchia *et al.* 2012) rather than pragmatic reasoning.

The Harnish–Levinson approach captures parts of the homogeneity and co-predication paradigms, but has some important empirical deficits. For this reason, much of this section is dedicated to modifying the theory. First, to capture the parts of the co-predication paradigm that the Harnish–Levinson account cannot obtain (as well as a number of independent datapoints around colour terms' quantificational force), I show in section 4.2 that the theory needs to claim that there is a constraint on the exhaustification of colour terms, such that exclusion must be computed ultra-locally to the adjective. I suggest as a possibility to derive such a constraint through an Agree relation between a^0 and Exh. Second, to capture heterogeneity-gaps, I suggest in section 4.3 to capture exhaustification through the trivalent exhaustivity operator of Bassi *et al.* (2021) rather than the standard Exh of Chierchia *et al.* (2012). With these two important changes to the Harnish–Levinson account, we can successfully capture the quantificational force of summative predicates.

Despite these improvements, I will also show in section 4.4 that the account straightforwardly cannot be extended to plural homogeneity. While my conclusion in this article will ultimately be that this is not a problem (subatomic and plural homogeneity could reasonably be united only in being locally computed exhaustification effects, as I will claim in section 7), this will nonetheless motivate the search for alternative accounts of subatomic homogeneity, which will be the focus of sections 5 and 6.

4.1 *Summative predicates exclude related predicates*

Harnish (1976) and Levinson (1983) focus on the paradigm in (51):

(51) a. The flag is green.
green $\approx$ *entirely green*
b. The flag is green and white.
green $\not\approx$ *entirely green*

In line with the discussion in section 3.2, these authors suggest that (51b) shows that the lexical meaning of colour terms is existential, not universal:[8]

(52) $[\![\text{green}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{green}(y)] \equiv \lambda x.\mathsf{green}_{\exists}(x)$.

From (52), Harnish and Levinson take (51a) to be the result of a quantity implicature. In a Gricean framework, (51a) can only be strengthened through the exclusion of stronger alternatives, in this case alternatives with conjoined colour terms ('The flag is green and white', and so on). Recent work (e.g., Katzir 2007, Fox and Katzir 2011) has suggested that alternatives cannot be more syntactically complex than the assertion, making this claim suspect. On the other hand, on the grammatical view of strengthening (e.g. Chierchia *et al.* 2012), it is often taken that Exh not only excludes stronger alternatives, but also non-weaker ones (e.g. Fox 2007):

(53) $[\![\text{Exh}_{\text{ALT}}(p)]\!] = 1$ iff $[\![p]\!] = 1$ and $\forall q \in \text{ALT}[[\![p]\!] \not\subseteq [\![q]\!] \rightarrow [\![q]\!] = 0]$

Thus, the Harnish–Levinson account can be recast in terms of Exh excluding alternatives formed through other bare (non-conjoined) colour terms:

(54) a. Exh_{ALT} [the flag is green].
b. ALT = {The flag is green, The flag is red, The flag is white, ...}
c. $[\![(54a)]\!] = 1$ iff the flag is green$_{\exists}$ $\wedge$ the flag is not white$_{\exists}$ $\wedge$ the flag is not red$_{\exists}$ $\wedge$ ...

The meaning in (54c) is that the flag has at least one green piece, and does not have a piece that is of any other colour. Given world knowledge that all areas of a surface must have a colour, this is pragmatically strengthened to mean that the flag is entirely green, in the same way that (55) with an overt *only* means that the flag is entirely green.

(55) The flag is only green.

It may be that the set of alternatives (54b) includes more than just sentences obtained by replacing *green* with other colour terms. Picture a window, part of which is made of green

8 Colour adjectives have quite rightly been described as degree predicates (e.g. Kennedy and McNally 2010). For instance, they can take comparative morphemes—e.g. one tree can be 'greener' than its neighbour, in the sense of having more green parts. This can be reconciled with (52). Indeed, the existential meaning in (52) could very well arise compositionally from the degree semantics of partial predicates (see Yoon 1996, Kennedy and McNally 2010, and Haslinger and Paillé 2023), if *green* is defined as 'having a degree of greenness greater than the scale minimum' (i.e., greater than zero).

stained glass, and the rest of which is entirely transparent. (56) is not a good description of such a window.

(56) The window is green.
(# *if half of the window is transparent*)

Given the truth conditions in (54c), this is not predicted, because it is indeed the case in (56) that the window has no colour other than green. Postulating *transparent* as part of the set of alternatives to *green* is of no help: given that green stained glass is still more or less transparent, we would then predict that (56) would be false/infelicitous of a window which was entirely green stained glass, contrary to fact. On the other hand, there is the adjective *clear* which implies colourless transparency. If *clear* is an alternative to *green* along with other colour terms (57), the oddness of (56) uttered of a half-green, half-clear window is captured: the window *does* have a clear part, but Exh excludes that the window has any clear parts.

(57) a. Exh_{ALT} [the window is green].
b. ALT = {The window is green, The window is red, The window is clear, ...}
c. $[\![(57a)]\!] = 1$ iff the window has a green part, does not have a clear part, and does not have a part of any colour other than green.

See Paillé 2023a for discussion of which kinds of predicates are alternatives to one another for exhaustification effects like the one being posited in this article (i.e. ultra-local ones on predicates, as described in section 4.2).

The Harnish–Levinson proposal immediately obtains the truth conditions of negative sentences (58), as non-exhaustified negations of the existential lexical meaning of colour terms. We will return to heterogeneity-gaps in negative sentences in section 4.3.

(58) The flag is not green.
≈ the flag is not green at all

What about co-predications? Let us start with the consistent ones, returning to the inconsistent ones in section 4.2.

In the case of conjoined colour terms, if there is a single Exh operator above both colour terms, existential-plus quantification is exactly what is predicted (59). In (59), I pretend that the only colour terms are green, white, and red, and assume that alternatives are obtained by replacing conjuncts by other colour terms and/or removing conjuncts entirely.

(59) a. Exh_{ALT} [the flag is green and white].
b. $\text{ALT} = \left\{\begin{array}{c}\text{the flag is green and white,}\\ \text{the flag is green and red,}\\ \text{the flag is red and white,}\\ \text{the flag is green,}\\ \text{the flag is white,}\\ \text{the flag is red}\end{array}\right\}$
c. $[\![(59a)]\!] = 1$ iff the flag is $\text{green}_\exists \wedge$ the flag is $\text{white}_\exists \wedge$ the flag is not $\text{red}_\exists$.

(59) means that the flag has a green part, has a white part, and does not have parts of any other colours. All pieces of the flag must therefore be green or white.

As for the co-predications with *also* (60), it turns out that there is already a literature claiming that *also* interacts with exhaustification.

(60) The white flag is also green.

Indeed, in empirically different examples like (61) where additive particles are also obligatory, it has independently been claimed that the additive's obligatoriness arises due to the interaction between additives and exhaustivity (Bade 2016, Aravind and Hackl 2017, and Paillé 2022a, following a similar claim by Krifka (1998) and Sæbø (2004)). On this view, *also* is required in (61) because without it, the second sentence would be exhaustified to mean that *only* Jade sang, contradicting the previous discourse. In (61), *Adam* and *Jade* bear alternatives due to being contrastive topics (Büring 2016).

(61) Adam sang. Jade #(also) sang.

Bade (2016) compares the hypothesis that *also* is obligatory in order to avoid unwanted exhaustification effects to an alternative view according to which they are obligatory whenever their presupposition is met (e.g. Chemla 2008, Singh 2008)—that is, due to the 'Maximize Presupposition' principle of Heim (1991). Since presuppositions project past negation, the Maximize Presupposition account predicts that additives would still be obligatory in negative sentences; but since exhaustivity standardly disappears under negation, the exhaustivity account predicts they should be optional (Bade 2016). The exhaustivity account is the one whose prediction is borne out; (62) does not need an additive (Bade 2016).

(62) Adam sang. Jade didn't (also) sing.

We conclude that additives interact with Exh in a non-trivial way; this means that (60) is expected from the Exclusion account of subatomic homogeneity.

Of course, one needs an account of *how* additives circumvent unwanted exhaustivity effects. This article is not the place to discuss the interaction between additives and exhaustivity at length. Let me simply follow the claim in Paillé 2022a that additives interact with Exh by pruning its alternatives (cf. Aravind and Hackl 2017). First, we will see in section 4.2 that (60) without *also* is best analysed with a local predicational Exh operator on each colour term:

(63) The [Exh_{ALT} white] flag is [Exh_{ALT} green].

I suggest that (60) is acceptable with *also* because, even though both colour terms are exhaustified independently of the other (i.e. the syntactic placement of Exh is no different than in (63)), when *also* is present, *white* is not an alternative to *green* and *green* is not an alternative to *white*. No contradiction arises.

(64) The [$\text{Exh}_{\{\text{white}, \sout{\text{green}}, \text{red}\}}$ white] flag is also [$\text{Exh}_{\{\sout{\text{white}}, \text{green}, \text{red}\}}$ green].

The flag is still entailed not to be of any other colour, so that the colour terms are existential-plus.[9]

9 Note that I have used the syntactic scope of Exh to explain the co-predications with *and*, and pruning of alternatives to explain co-predications with *also*. Having a different explanation for each 'conjunctive'

Despite these successes, there are aspects of colour terms' quantificational force that the Exclusion theory, as it stands, does not capture. This includes a variety of data where colour terms are more consistently universal than predicated by the Harnish–Levinson account (including the inconsistent co-predications), and the classic observation (e.g. Löbner 2000) that colour terms lead to a truth-value gap in non-homogeneous situations. I take up these points in sections 4.2 and 4.3 respectively.

4.2 *The exclusion of colour terms is always computed locally*

If the Exclusion account is to be maintained, it must come with the stipulation that Exh is always local to the colour term. In all cases where a difference in meaning is predicted to be intuited according to whether Exh takes scope globally or locally to the colour term, the meanings that would be obtained from a global Exh are not intuited. Most of the examples in this section involve monoclausal sentences, so 'locally' has stronger meaning than just 'in the same clause'. For our purposes, it will suffice to understand this locality constraint as Exh necessarily being in the maximal projection (*a*P) of the colour term. Such an 'ultra-local' Exh is shown in (65), putting aside the question of whether Exh is necessarily ultra-local in this particular example (where local or global Exh operators would produce the same meaning):

(65) $[\![$The flag is $[_{\text{AP}}$ Exh$_{\text{ALT}}$ green$]]\!] = 1$ iff the flag is [green$_\exists$ & not red$_\exists$ & not white$_\exists$].

(65) has an Exh operator taking a colour term, and nothing else, as its prejacent—despite Exh having been defined as a propositional operator. I therefore define a predicational Pred-Exh operator (cf. Mayr 2015), based on a generalized notion of entailment. A predicate P entails another predicate Q if for all x, $P(x)$ entails $Q(x)$. For instance, *dog* entails *animal*, and *scarlet* entails *red*.

(66) $[\![\text{Pred-Exh}_{\text{ALT}}(P)]\!] = \lambda x.[\![P]\!](x) \wedge \forall Q \in \text{ALT}[[\![P]\!] \not\subseteq [\![Q]\!] \rightarrow [\![Q]\!](x) = 0]$.

Thus:

(67) $[\![\text{Pred-Exh}_{\text{ALT}}\ \text{green}]\!] = \lambda x.\mathsf{green}_\exists(x) \wedge \neg\mathsf{white}_\exists(x) \wedge \neg\mathsf{red}_\exists(x) \wedge \ldots$

I will suggest in section 4.2.3 to derive the locality constraint by having colour terms Agree with Exh; since there is no upward Agree (Chomsky 2001), Exh must be in the colour term's projection. First, however, let us focus on the empirical observations about the locality of Exh.

expression is motivated from the different empirical behaviour of these two expressions: while *also* can solve the problems stemming from exhaustification at a distance (i.e., across clauses), *and* cannot:

(i) a. The flag is white. It is also green.
b. 1) #The flag is white and it is green.
2) The flag is white and green.

Further, if (63) is the right LF for the sentence without *also*, it would be impossible for a single *also* to take scope below each Exh to add the other colour term's meaning as an entailment. Scope is therefore a non-starter to explain why *also* makes co-predication possible.

4.2.1 *Inconsistent co-predications: initial motivation for a constraint on Exh's syntax*
The initial motivation for a locality constraint comes from the half of the co-predication paradigm that the Exclusion theory, as stated, cannot explain: inconsistent examples like (68).

(68) a. #The white flag is green.
b. #The white green flag is high.

Let us start with the the predicational colour term *green* in (68a), returning below to colour terms internal to definite descriptions; for now, pretend for the sake of argument that only *green* has alternatives in (68a), not *white*. If Exh could take scope anywhere, it would be possible for Exh to scope globally (69). From this global position, Exh's prejacent entails both the whiteness and greenness of the flag. Since Exh does not exclude alternatives that are entailed by its prejacent, neither colour is excluded, and no contradiction results:

(69) a. Exh_{ALT} [The white flag is green].
b. ALT = {that the white flag is P : P is a colour adjective}
c. $[\![(69a)]\!] = 1$ iff the $\text{white}_{\exists}$ flag is $\text{green}_{\exists} \wedge \neg$[the $\text{white}_{\exists}$ flag is $\text{red}_{\exists}$].
$\Rightarrow$ no contradiction

In order for *green* in (68a) to be strengthened irrespective of *white*, Exh must necessarily appear locally to it, as in (70) (where I continue to overlook that *white* presumably has alternatives and a local Exh too):

(70) a. The white flag is [Pred-Exh_{ALT} green].
b. ALT = {P : P is a colour adjective}
c. $[\![(70a)]\!] = 1$ iff the $\text{white}_{\exists}$ flag is [$\text{green}_{\exists}$ & not $\text{white}_{\exists}$ & not $\text{red}_{\exists}$].
$\approx$ the $\text{white}_{\exists}$ flag is only $\text{green}_{\exists}$
$\Rightarrow$ contradiction

In fact, the need for an ultra-local Exh is only motivated on the assumption that Exh can see entailment relations among colour adjectives, and therefore knows what it can and cannot exclude. To obtain a contradiction in (69) without forcing Exh to be ultra-local, one could try to claim that Exh is so blind to the content of predicates that it does not know that *The* $\textit{white}_{\exists}$ *flag is* $\textit{white}_{\exists}$ is entailed by *The* $\textit{white}_{\exists}$ *flag is* $\textit{green}_{\exists}$. Exh would therefore negate *The* $\textit{white}_{\exists}$ *flag is* $\textit{white}_{\exists}$, obtaining the semantic deviance. But there is independent evidence that Exh does know which colour predicates can be excluded.[10] Consider the following minimal pair:

10 This claim is not incompatible with the claims by Magri (2009) that Exh does not access world knowledge, at least if a distinction is made between world knowledge and the 'core' conceptual meaning of predicates. Magri's claim is that Exh does not take world knowledge into consideration, so that sentences like (i) are strengthened even if we know that all people in a given nation-state live under the same government.

(i) #Some inhabitants of Japan live in a constitutional monarchy.

While it follows from this that Exh strengthens sentences without taking general world knowledge into account, it does not follow from this that Exh is blind to the core conceptual–lexical meaning of

(71) a. #The red flag is green.
b. The red flag is scarlet.

In (71b), *scarlet* does not exclude *red* because it entails it. Crucially, while *red* is a basic-level colour term and *scarlet* is a subordinate-level term, we know that basic-level colour terms are alternatives for subordinate-level colour terms from data like (72):

(72) The flag is scarlet.
≈ the flag is entirely scarlet

If *scarlet* only had subordinate-level colour terms as alternatives, (72) would not be universal on the Exclusion theory. Only excluding subordinate-level colour adjectives would not actually exclude the entire colour wheel; English and other languages cover the entire colour wheel with their basic colour terminology (Berlin and Kay 1969), but their subordinate colour terms are more of a patchwork. For instance, in my English, I have very few subordinate colour terms for types of pink or orange (and most of them are structurally complex, like *hot pink*), and they certainly do not cover all hues of pink/orange. (72) would therefore be compatible with the flag having pink and orange parts if *scarlet* only excluded subordinate-level colour terms. Thus, in (71b), we can infer that *scarlet* has basic colour terms including *red* as alternatives. But Exh apparently does not exclude *red*; it must therefore be that Exh sees entailment relations among colour predicates.[11] Hence, the only way to obtain clause-internal contradictions is through a locality constraint on Exh.

Let us now turn to the attributive adjectives in both (68a) and (68b). Attributive colour terms must be exhaustified just as much as predicative ones; otherwise, (68b) would be consistent. But these too must be subject to a locality constraint. Quite outrageously, with a colour adjective that is part of a definite description, a global Exh would create entailments about other flags; (73) shows this for (68b), but the same goes for (68a).

(73) a. Exh_{ALT} [The white green flag is high].
b. ALT = {The white green flag is high, The red blue flag is high, …}
c. $[\![(73a)]\!] = 1$ iff the white$_\exists$ green$_\exists$ flag is high $\wedge$ the red$_\exists$ blue$_\exists$ flag is not high $\wedge$ …

Note that the truth conditions in (73c) are not aligned with intuitions in two ways: there are entailments about other flags, and no contradiction is created. On the other hand, the observed meaning can be created by having an Exh operator local to each attributive colour term:

(74) a. The [Pred-Exh_{ALT} white] [Pred-Exh_{ALT} green] flag is high.
b. ALT = {white, green, red, …}

predicates. Exh in (i) might well have access to the meanings of words like *inhabitants* or *monarchy*; it simply does not take into account independently known facts of the world.

11 As for why *red* does not exclude *scarlet*, the simplest assumption is that *scarlet* is not an alternative to *red* because *red* is a basic-level colour term while *scarlet* is a subordinate-level colour term. Basic colour terms are alternatives to subordinate ones, but not vice-versa.

c. $[\![(74a)]\!] = 1$ iff the flag that is $\begin{pmatrix} \text{white}_\exists \ \& \\ \text{not green}_\exists \ \& \\ \text{not red}_\exists \end{pmatrix}$ and $\begin{pmatrix} \text{green}_\exists \ \& \\ \text{not white}_\exists \ \& \\ \text{not red}_\exists \end{pmatrix}$ is high.

$\Rightarrow$ contradiction

(74c) is contradictory and lacks entailments about other flags, as desired.

The claim that colour terms constrain Exh's syntax may come across as a post-hoc complication only posited to capture the inconsistent co-predications. However, in the following subsection, I show that the locality constraint is observed even outside of inconsistent co-predications. Following that, I will show that the constraint can simply be modelled as an Agree relation between colour terms and Exh.

4.2.2 *More evidence for a locality constraint on Exh* On the Exclusion account, the need to constrain Exh's syntax is general. When a colour term co-occurs with another scope-bearing element, Exh must scope locally to the colour term. Intuitively, (75) means that exactly one flag is entirely green.

(75) Exactly one flag is green.
$\approx$ exactly one flag is entirely green

But a global Exh, scoping above both *exactly one* and the colour term, does not actually create this strong meaning:

(76) $[\![\text{Exh}_{\text{ALT}} \text{ [exactly one flag is green]}]\!] = 1$ iff $\begin{cases} \text{exactly one flag is green}_\exists \wedge \\ \neg[\text{exactly one flag is white}_\exists] \wedge \\ \neg[\text{exactly one flag is red}_\exists] \end{cases}$

In (76), *green* is not universal; it means that there is exactly one flag with any green on it.[12] This problem is avoided by having Exh scope below *exactly one flag*—that is, ultra-locally to the colour term:

(77) $[\![\text{Exactly one flag is [Pred-Exh}_{\text{ALT}} \text{ green]}]\!]$
$= 1$ iff exactly one flag is [green$_\exists$ & not white$_\exists$ & not red$_\exists$].
$\approx$ exactly one flag is entirely green

The same goes to capture the meaning of colour terms in downward-entailing (DE) environments. Exhaustivity computed below a DE operator leads to global weakening, and is therefore strongly dispreferred (e.g. Chierchia *et al.* 2012, Fox and Spector 2018). This is what leads Križ and Spector (2021) to criticize exhaustivity accounts of homogeneity: neither plural nor subatomic homogeneity disappears in DE environments:

12 As stated, (76) also has the problem that there is a non-intuited entailment about the number of flags that are partly of other colours. But this problem would be resolved by having alternatives obtained by replacing *one* with other numerals ({exactly zero flags are white$_\exists$, exactly one flag is white$_\exists$, exactly two flags are white$_\exists$, ...}). The alternatives from colour terms other than the asserted *green* are not innocently excludable (Fox 2007), since it cannot be that there is no numeral n such that exactly n flags are partly of such-or-such colour.

(78) a. If the flag is pink, you should jump around.
≈ 'If the flag is **all** pink, you should jump around.'
b. If you solve the problems, you will pass the exam. (Križ 2015:27)
≈ 'If you solve **all** the problems, you will pass the exam.'

Putting aside the plural for now, (78a) is in fact far from unexpected given that we have already observed other instances where colour terms, even outside of DE environments, require a local Exh. The right meaning for (78a) is obtained by placing an Exh operator below *if*.

To be sure, neither summative predicates nor plural predication are consistently interpreted as universal below *if*, as in (79) (cf. Križ 2015:27 for similar examples with plural predication):

(79) a. Scenario: *We need to arrange a shipment of entirely white flags. There is a pile of flags, most of which are entirely white but a few of which are both white and green. An employee is told:*
b. We want to only have white flags over here, so ...
(i) ...if you see a white flag you should keep it.
(≈ 'if you see an entirely white flag, ...')
(ii) ...if you see a green flag you should throw it away.
(≈ 'if you see a partly green flag, ...')

A possible way to analyse this is to claim that Exh is non-local or entirely absent in the non-maximal (79b-ii); this line of reasoning would rely on the presence of non-maximality below *if* as evidence that Exh is not *necessarily* present locally with summative predicates. Naturally, nothing forces such an analysis, because in section 2, we already posited mechanisms obtaining non-maximality that are independent of Exh's presence/absence or its locality (see also section 4.3 below).[13]

There are two issues with the view that non-maximality under *if* shows that Exh is scopally flexible. The most important is that it cannot explain why (78a) is most readily intuited out-of-the-blue as involving a universal below *if*; the standard view is that Exh is dispreferred below DE operators, so without a locality contraint on Exh, an existential meaning under *if* is expected to be the default reading. A universal reading would have to be motivated by discourse context. But what we observe in (78a) and (79b) is the opposite: the universal is default, while the non-maximal reading requires an explicit scenario.

The second, weaker issue with the view that non-maximality should be analysed as evidence for flexible exhaustification is that not all cases of non-maximality (whether under *if* or elsewhere) are purely existential like (79b-ii). Many instances of non-maximality require a local Exh together with a local application of Križ's mechanism for non-maximality. For example:

(80) a. Scenario: *You must dress more or less entirely in white to access an upper-class tennis court, but the court is not particularly strict. Someone tells you:*

13 I follow Križ (2015) in assuming that his mechanism can obtain non-maximality in embedded environments; see his ch. 3 for discussion.

b. If your clothes are white, they'll let you in. You'll be fine as long as other colours don't cover more than a few square inches.
≈ 'if your clothes are at least mostly white, …'

Here, we have non-maximality below *if*, with Exh still necessarily appearing below *if*, since the meaning is stronger than existential. Since non-maximality can often be shown to involve a local Exh (80), it is appealing to analyse the existential non-maximality in (79b-ii) in the same terms: a local Exh with local non-maximality.

We conclude that the quantificational force of summative predicates is always computed locally. One apparent exception to this comes from sentential negation; we return to this issue in section 4.3.

4.2.3 *Formalizing Exh's locality constraint as an Agree relation* Is this locality requirement just a stipulation? It would certainly be hard not to view it this way if it was only found with colour terms or summative predicates. In section 7 (see also Paillé 2022b), I suggest that Exh is generally found locally with predicates. I do not elaborate this idea in the present paper, but what I suggest is that language does not just exhaustify clauses/propositions, but also predicates themselves, independently of the clauses they are in. Postulating the existence of predicate-strengthening is not inherently more stipulative than postulating the (well-established) existence of sentence-strengthening.

Since the purpose of this article is to compare different theories of the quantificational force of summative predicates, I will not dwell too long on what causes the locality constraint, leaving this for other work. I will simply show as a proof of concept that the locality can be modelled through a well-established syntactic mechanism. What I tentatively suggest is that colour terms (or rather: the derivational morpheme a^0 that merges the roots into the syntax) Agree with Exh; they therefore require an Exh operator to be present in the clause and specifically within their *a*P. This is essentially taking a proposal by Chierchia (2013), according to which Exh Agrees with alternative-triggering expressions, and turning it on its head; what we have here is an alternative-triggering expression (or the morpheme selecting it) Agreeing with Exh.

To start with, a theory of the locality constraint based on a syntactic interaction between Exh and a^0 cannot be built around selection. Selection is an appealing mechanism for data like (81), where each colour terms has its own Exh (within their *a*P, I assume, although that's not the only logical possibility).

(81) #The white green flag is high.

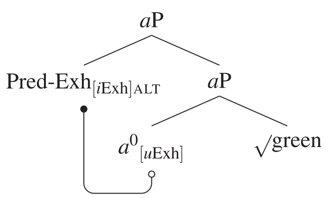
But other data involve a single Exh for more than one colour term, which a theory built around selection would not be able to capture. This is the case for (82), where a single Exh must scope above both colour terms.

(82) The flag is white and green.

Colour terms therefore cannot be taken to select Exh, in which case each would have its own Exh in (82). But if they merely Agree with Exh, the contrast between (81) and (82) can be captured.

In the basic (non–co-predicational) case, we can derive the locality constraint via Agree as follows. I follow the theory of Agree in Chomsky 2001, but augment it with Spec–Head agreement (see e.g. Clemens and Coon 2018). On this approach, a^0 bears a [*u*Exh] feature which probes for an expression bearing an [*i*Exh] feature, which in most cases will be Exh.[14] There is no truly upward Agree, where a head H Agrees with an expression above its maximal projection HP; but there *is* agreement within the HP, including Spec–Head agreement. So if Exh is adjoined to H (here, a^0), Agree can proceed (83) and the derivation converges syntactically.

(83)



As for the conjunction data, the Agree theory derives that a single Exh can scope above both colour terms due to the classic proposal that conjoined phrases inherit the category of the conjuncts (rather than projecting as a ConjP). In our case, conjoining two *a*Ps creates a bigger *a*P (84). This is in fact predicted if the head *and* is acategorial; according to Chomsky (1995), acategorial roots do not project (see Paillé 2022b:ch. 6 for more discussion).

14 Presumably, the goal of agreement could also be an overt *only*. Consider (i):

(i) The flag is only green.
≈ the flag has no colour other than green

(i) presupposes that the flag has a green part and asserts that for all other colours, there is no part of that colour. If there was an Exh below *only*, however, *only*'s prejacent would entail that the flag is green and no other colour. (i) would therefore presuppose that the flag has no colour other than green, making *only*'s assertive contribution vacuous. This issue is avoided if there is simply no Exh in (i), which can be captured if the [*u*Exh] probe can Agree with *only*.

Of course, we now predict a locality constraint on *only* identical to the one posited in the main text for Exh. This is borne out. Outside of the domain of colour terms, *only* can arbitrarily appear in an 'it is the case' clause above its focus associate:

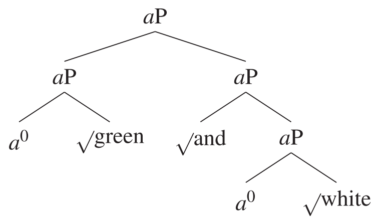
(ii) a. Aisha only saw [the children]$_F$.
b. It is only the case that Aisha saw [the children]$_F$.

In contrast, *only* must surface near its associate if it is a colour term:

(iii) a. The flag is only [green]$_F$.
b. ??It is only the case that the flag is [green]$_F$.

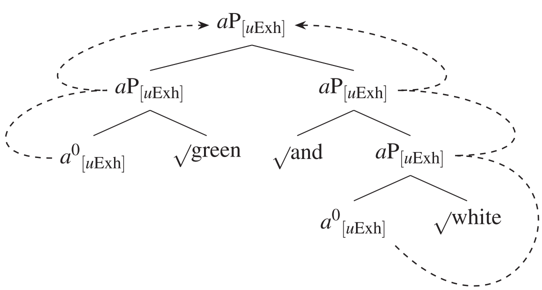
In the right pragmatic context, (iiib) could be used to communicate that it's only the case that the flag is green and not some other property like being high or torn, but—in contrast to (iiia)—it cannot be used to mean merely that the flag is entirely green.

(84)



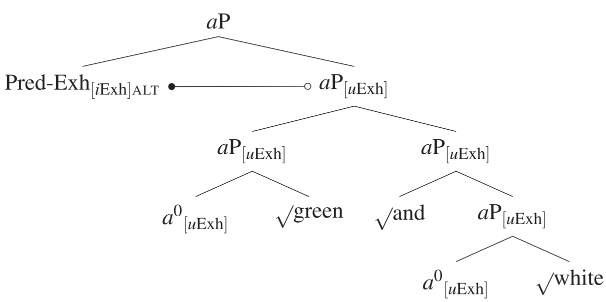
If the [*u*Exh] feature on each a^0 in (84) projects together with the category label, we expect the entire phrase to only require one Exh. In (85), feature projection (presumably happening in tandem with category projection) is shown with a dotted arrow.

(85)



From there, Exh is merged and Agree takes place between the $a\mathrm{P}_{[u\mathrm{Exh}]}$ label and Exh (see Béjar and Rezac 2009) on Agree from labels):

(86)



The [*u*Exh] feature is taken care of, and the syntax converges.

In sum, Exh's locality constraint with colour terms can be derived through an Agree relation between a^0 and Exh. Given that Exh is an operator present in the syntax, there is no reason for it not to engage in operations like Agree. In section 7 and Paillé 2022b, I suggest that this does not only occur with colour terms and other summative predicates, but with predicates generally.

4.2.4 *Section summary* In this subsection, I have modified the Exclusion account of Harnish (1976) and Levinson (1983) by placing a locality constraint on the Exh operators associating with colour terms, as required to capture not only the inconsistent half of the co-predication paradigm, but also a number of independent datapoints where colour terms are interpreted as universal more consistently than predicted by Harnish and Levinson. I suggested to model this constraint through an Agree relation whereby colour terms (or a^0) probe for Exh, ensuring that the syntax only converges if there is an Exh operator within the colour term's maximal projection.[15]

4.3 *Deriving heterogeneity-gaps through Pexh*

We now turn to a second modification of the Harnish–Levinson Exclusion account. As spelled out so far, the Exclusion theory does not predict heterogeneity-gaps as in (87).

(87) $$[\![\text{The flag is green}]\!] = \begin{cases} 1, \text{ iff the flag is all green;} \\ 0, \text{ iff the flag is not green at all;} \\ \#, \text{ otherwise} \end{cases}$$

However, as stated, the Exclusion theory predicts that positive sentences like (87) would in fact be *false* in non-homogeneous cases: (88) is expected to be true if the flag is green and no colour, and to be false otherwise.[16]

(88) Exh_{ALT} [the flag is green].

Can the Exclusion theory obtain the heterogeneity-gap in (87)?

It can, with one change: we need to follow Bassi *et al.* (2021) in taking Exh to be trivalent. Due to different empirical concerns entirely, Bassi *et al.* (2021) argue that semantic exhaustification only affects truth conditions, not falsity conditions. They suggest the trivalent operator in (90) as a substitution for the bivalent Exh repeated from (53) in (89).

15 A reviewer asks if the suggested relation between Exh and a^0 is compatible with (i), which is intuited as consistent while having epistemic modal adverbs within the conjuncts:

(i) The flag is certainly white and certainly green. (It is certainly not red.)

On the proposal in this article, the fact that (i) is non-contradictory means that there is a single Exh above both *white* and *green*:

(ii) [$_{a\text{P}}$ Pred-Exh_{ALT} [$_{a\text{P}}$ certainly white and certainly green]]

For (ii) to hold, it must be possible for epistemic modal adverbs like *certainly* to be within an *a*P and therefore take subpropositional scope. In fact, this possibility has been noted in the literature (see Bogal-Allbritten 2013, Condoravdi *et al.* 2019, and citations therein); Bogal-Allbritten (2013) gives examples including:

(iii) a. They found a [$_{a\text{P}}$ probably precancerous] mole on John's back.
b. Mary ate [$_{\text{DP}}$ possibly the most expensive pizza in Amherst].

Let me also give an example found online where a token of *certainly* modifying a colour adjective is clearly subpropositional due to being within a definite DP:

(iv) Now I use a hard black brush to paint in the parts of the sky that are certainly black, and a white brush to paint over **the certainly white parts**. That way I only have to tweak the parts where white and black join. (https://www.dpreview.com/forums/post/42080511; accessed Oct. 23, 2023)

As such, (i) is not worrisome for the claim that a colour term's Exh operator must be within its *a*P.

16 For simplicity of presentation, I do not place the exhaustivity operators ultra-local to the colour terms in this section; nothing hinges on this.

(89) $[\![\text{Exh}_{\text{ALT}}(p)]\!] = 1$ iff $[\![p]\!] = 1$ and $\forall q \in \text{ALT}[[\![p]\!] \not\subseteq [\![q]\!] \rightarrow [\![q]\!] = 0]$

(90) $$[\![\text{Pexh}_{\text{ALT}}(p)]\!] = \begin{cases} 1, \text{iff } [\![p]\!] = 1 \text{ and } \forall q \in \text{ALT}[[\![p]\!] \not\subseteq [\![q]\!] \rightarrow [\![q]\!] = 0]; \\ 0, \text{iff } [\![p]\!] = 0; \\ \#, \text{otherwise} \end{cases}$$

A sentence exhaustified with Pexh is true if the prejacent is true and all the non-entailed alternatives are false. But the falsity conditions depend only on the prejacent.[17]

By replacing Exh with Pexh, the Exclusion theory obtains heterogeneity-gaps:

(91) $$[\![\text{Pexh}_{\text{ALT}} \text{ [the flag is green]}]\!] = \begin{cases} 1, \text{iff } \mathsf{green}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f); \\ 0, \text{iff } \neg\mathsf{green}_{\exists}(f); \\ \#, \text{otherwise} \end{cases}$$

Consider the logical value obtained if the flag is half green, half white. In such a case, it is neither the case that $\neg\mathsf{white}_{\exists}(f)$ holds (as would be needed for the sentence to be true), nor that $\neg\mathsf{green}_{\exists}(f)$ holds (as would be needed for the sentence to be false). Therefore, the sentence is undefined, as desired.

There is another benefit to adopting the trivalent exhaustivity operator. On the face of it, the truth condition of negative sentences can be obtained on the Exclusion account by stating that such sentences are not exhaustified at all. However, in section 4.2, I claimed that there is always a local Exh with colour terms, even if this leads to global weakening. This makes it unclear why sentential negation would be the exception to this. On the particular theory of locality tentatively suggested above (based on Agree), Exh is strongly predicted to always be obligatory and necessarily local with colour terms, even under negation. But putting a bivalent Exh operator below negation does not lead to the intuited meaning:

(92) $[\![\text{not [Exh}_{\text{ALT}} \text{ [the flag is green]]}]\!] = 1$ iff
$\neg[\mathsf{green}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f)]$
$\equiv \neg\mathsf{green}_{\exists}(f) \vee \mathsf{red}_{\exists}(f) \vee \mathsf{white}_{\exists}(f)$

The truth conditions in (92) do not actually require the flag not to have any green parts. With Pexh, however, nothing goes wrong if exhaustivity is computed under negation. Pexh only affects truth conditions, not falsity conditions—and *not* reverses these.

(93) $$[\![\text{not}(p)]\!] = \begin{cases} 1, \text{iff } [\![p]\!] = 0 \\ 0, \text{iff } [\![p]\!] = 1 \\ \#, \text{otherwise} \end{cases}$$

Thus, as far as the truth conditions are concerned, Pexh is vacuous under *not*. We can therefore claim, following the logic of section 4.2, that Pexh *is* present under negation:

(94) a. $$[\![\text{Pexh}_{\text{ALT}} \text{ [the flag is green]}]\!] = \begin{cases} 1, \text{iff } \mathsf{green}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f) \\ 0, \text{iff } \neg\mathsf{green}_{\exists}(f) \\ \#, \text{otherwise} \end{cases}$$

17 Interested readers are referred to these authors' work for empirical movitation for this as well as discussion of how this fits with intuitions for exhaustified sentences. The operator is called Pexh because it is a 'presuppositional' Exh.

b. $[\![\text{not [Pexh}_{\text{ALT}}\text{ [the flag is green]]}]\!] = \begin{cases} 1, \text{iff } \neg\mathsf{green}_{\exists}(f) \\ 0, \text{iff } \mathsf{green}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f) \\ \#, \text{otherwise} \end{cases}$

Note that Pexh still affects the falsity (and undefinedness) conditions, so it is not entirely vacuous. This is as desired, since summative predicates can lead to heterogeneity-gaps (and non-maximality) in both positive and negative sentences (see Paillé 2023b for more discussion of non-minimality).

Pexh therefore allows the Exclusion account to both capture heterogeneity-gaps, and capture that negation behaves differently from other DE environments in lacking a strengthening (globally weakening) effect in the truth conditions. While I adopt the Pexh operator of Bassi *et al.* (2021), I will continue writing Exh and having bivalent truth conditions in the rest of this article.

On the surface, the very properties that have just led to the adoption of Pexh may appear to problematize the proposal that trivalent Pexh should replace bivalent Exh: Pexh is proposed by Bassi *et al.* (2021) to compute scalar implicatures, but these have not been described as having truth-value gaps, for example. In fact, postulating truth-value gaps for scalar implicatures might not be undesirable. On the pragmatic theory of strengthening via Gricean implicatures, utterances come with a theoretically substantial distinction between their 'assertion' and their 'implicatures'. This makes it possible to describe sentences as having a true assertion but a false implicature, and therefore (on the whole) being neither true nor false. The semantic theory of exhaustification treats implicatures as part of the assertion, however, and therefore (with a bivalent Exh) cannot make this distinction; if Adam ate all of the cookies, (95) (with Exh) would be simply false.

(95) Adam ate some of the cookies. # if he ate all of them

But Pexh makes it possible to capture this kind of under-informativity—with Pexh, (95) is expected to be neither true nor false if Adam ate all the cookies.[18] Note that, for our purposes, the presuppositional status of the undefinedness stemming from Pexh is not what matters (see section 6 of Bassi *et al.* 2021 for some discussion).

Capturing heterogeneity-gaps through Pexh immediately leads to the expectation of observing non-maximality with colour predication, assuming Križ's (2015) theory of non-maximality described in section 2. Let me briefly comment on why this is preferable on the Exclusion theory to an alternative way to weaken utterances, namely by excluding fewer alternatives ('pruning' them), as Bar-Lev (2021) suggests to obtain non-maximality in his Inclusion theory (which I discuss in section 5.2). Let us first consider a sentence displaying non-maximality:

(96) a. Scenario: *In the fall, you pick up a leaf that is mostly orange, but also partly green and brown. You say:*
b. The leaf is orange.

Hypothetically, this non-maximality could be obtained by pruning alternatives with *green* or *brown*:

18 This is not the only way to analyse (95) on the semantic theory; its oddness could instead come about from the sentence being true on a parse without Exh but false on a parse with Exh (cf. Bar-Lev 2021).

(97) a. ALT = {The leaf is orange$_\exists$, The leaf is pink$_\exists$, ~~The leaf is green$_\exists$~~, ~~The leaf is brown$_\exists$~~, The leaf is blue$_\exists$, ...}
b. ⟦Exh$_{ALT}$ [(96b)]⟧ = 1 iff the leaf is orange$_\exists$, maybe green$_\exists$ or brown$_\exists$, and no other colour

However, on the Exclusion theory, the 'pruning' path to non-maximality differs from Križ's empirically, since the pruning approach can only create existential force for colour terms. (97b) means that the leaf has an orange part, but this part may be very small and the leaf might be mostly green or brown. In contrast, Križ's approach can distinguish between many different quantificational forces, predicting that non-maximality can be stronger than mere existential meaning. For this reason, pruning cannot be all there is to non-maximality: there are some clear cases where non-maximal sentences are more than existential, including (98) as well as e.g. (7) and (80).

(98) a. SCENARIO: *For a temporary art installation, you are making a large mosaic using leaves. There's a part of the drawing that should all be solid orange, but this part is still missing a lot of leaves. People will be looking at the mosaic from quite a distance to appreciate it as a drawing, so it's okay if the leaves you find are not actually fully orange.*
b. This leaf is orange.
⇒ FELICITOUS for a leaf that is mostly orange, with some green/brown
⇒ INFELICITOUS for a leaf that is mostly green/brown, with some orange

On the Exclusion theory of subatomic homogeneity, (98) requires Križ's theory, and therefore the heterogeneity-gaps created by Pexh.[19]

This concludes my modifications to the Harnish–Levinson proposal. On my Exclusion account, the assertion of a lexically existential colour term is intuited as universal because it involves the exclusion of other colour terms. This exclusion is necessarily computed locally, because colour terms Agree with Exh. The Exh operator is trivalent, resulting in heterogeneity-gaps.

4.4 *No extension of the Exclusion account to plural homogeneity*

The rest of this article will focus on showing that various theories of plural homogeneity cannot be extended to explain the quantificational force of summative predicates, due to the data from co-predications. In this section, I do the opposite, showing that the Exclusion account of subatomic homogeneity cannot be extended to plural homogeneity.

The Exclusion account crucially relies on world knowledge, so that 'having a green part, and not having a part of any other colour (or a clear part)' is strengthened to meaning 'entirely green'. But this cannot carry over to plural homogeneity; world knowledge does

19 Bar-Lev (2021) suggests that non-maximality with positive sentences is easier to obtain than non-minimality in negative sentences. If this observation carries over to summative predicates (see Paillé 2023b), one could try to capture this on the Exclusion theory by stating that pruning is available in addition to Križ's mechanism. The idea would be that both non-maximality and non-minimality can be obtained by Križ's mechanism, but pruning can only affect the truth conditions of positive sentences—it affects the *falsity* conditions of negative sentences. This makes weakening through pruning easy to detect only in positive sentences.

not dictate anything about the parts of pluralities. Attempting to capture plural homogeneity through the exclusion of related predicates would look as in (99) (where, crucially, *singing* is not intonationally/contrastively focused). In (99), I am marking *singing* as 'existential' to create a parallel with my account with colour terms. We can put aside how this arises compositionally (see section 5.2 for Bar-Lev's (2021) proposal); what matters is simply that the non-exhaustified meaning of *the children are singing* is that at least one of them is singing, in the same way that the non-exhaustified meaning of *the flag is green* is that at least one part of the flag is green. On the Exclusion account as extended to plural homogeneity, this would then be exhaustified to exclude conceptually related predicates:

(99) $$[\![\text{Exh}_{\text{ALT}}\ [\text{the children are singing}_{\exists}]]\!] = 1 \text{ iff } \begin{cases} \text{there is a child who is singing } \wedge \\ \text{there is no child who is dancing } \wedge \\ \text{there is no child who is talking } \wedge \\ \text{there is no child who is } \ldots \end{cases}$$

There are two problems here. First, unlike the subatomic parts of surfaces which must all have a colour, it is in fact possible for individuals to do nothing. World knowledge does not make (99) entail that all children are singing. The utterance means that at least some children are singing and no children are doing anything else—but as far as world knowledge is concerned, there could be children who are *neither* singing *nor* doing anything else. Second, the entailments about children not doing other things are not in fact intuited without contrastive focus on *singing*. The children could be both singing and dancing, for example.

To salvage this, one could try to constrain the alternatives in (99) to only being mutually exclusive predicates (that is: mutually exclusive sentences obtained by replacing the verb with inconsistent predicates); *sing* would not have *dance* as an alternative, but it would have *talk* and *keep quiet*, for example. If at least one child in the plurality is singing, none is talking, and none is keeping quiet, they must all be singing. However, on this proposal, we lose the ability to explain the *subatomic* homogeneity data, where alternatives are crucially *consistent* predicates—for colour terms, they are lexically *existential* colour terms. If only *inconsistent* predicates could create alternatives, colour terms would not meet this requirement, and would therefore not be alternatives to one another. To defend this alternative view, one would need to explain why this mutual-exclusivity requirement among alternatives would hold for plural homogeneity but not subatomic homogeneity. There is also the problem that, if we take the view that alternatives are no more syntactically complex than the assertion (Katzir 2007; Fox and Katzir 2011), *keep quiet* cannot be an alternative to *sing*—so nothing ends up forcing the meaning that more than one child is singing.

In sum, the Exclusion account of subatomic homogeneity does not extend to plural homogeneity.

4.5 *Section summary*

Harnish (1976) and Levinson (1983) propose that colour terms are lexically existential, but are intuited as universal in positive sentences because their assertion involves the exclusion of other colour terms. In this section, I reformulated their account according to the grammatical theory of strengthening of Chierchia *et al.* (2012), and modified it in two ways. First, to capture the quantificational force of colour terms across a variety of examples, there must be a constraint on the syntax of Exh such that it necessarily occurs locally to the colour term. Second, to capture that non-homogeneous cases are undefined rather than false, I suggested to use the trivalent exhaustivity operator of Bassi *et al.* (2021) rather than the standard bivalent operator. Let us recap the ingredients of this approach and why they are necessary:

(100) The Exclusion account
 a. Summative predicates are lexically weak.
  ⇒ This obtains the truth conditions of negative sentences.
 b. They are exhaustified to exclude other same-class summative predicates.
  ⇒ This obtains 'universal' (more accurately: 'exclusive') and existential-plus meanings.
 c. The exhaustification is local, but with some flexibility (as modelled through Agree).
  ⇒ This captures that some co-predications are intuited as contradictions, while conjunctions are consistent.
 d. This exhaustification can be partly obviated by *also*, which prunes alternatives.
 e. The exhaustivity operator is trivalent.
  ⇒ This captures heterogeneity-gaps (and negative sentences, if the operator must appear below negation, as predicted by the Agree model of locality).

An important feature of this account, of course, is that nothing creates *semantically* universal meaning for summative predicates, in any sentence.

Despite all this work, we saw that the theory goes nowhere for plural homogeneity. This is not necessarily undesirable: the subatomic and plural homogeneity paradigms could have some common cause while differing in other ways. They could both be exhaustification effects but involve different kinds of alternatives, for example. Nevertheless, we now turn to considering other theories of plural homogeneity and seeing how they fare in the subatomic domain, with particular attention to the co-predication paradigm. In section 5, I focus on theories positing semantically universal meaning for positive sentences. These create too many contradictions; they cannot capture the existential-plus meaning of summative predicates in consistent co-predications. Then, in section 6, I focus on underspecification accounts of plural homogeneity. These create too few contradictions; they predict all co-predications to be consistent. Thus, none of the other theories of homogeneity can capture the quantificational force of summative predicates across-the-board, giving significant weight to the Exclusion account.

5. THEORIES OF HOMOGENEITY THAT CREATE TOO MANY CONTRADICTIONS

In this section, we consider two alternative theories of subatomic homogeneity, both of which derive universal quantification for positive sentences in the semantics and wrongly predict all co-predications to be inconsistent; I will focus on conjunctions like (101), which I showed in section 3 to be Boolean.

(101) The flag is white and green.

The first theory is the classic suggestion that summative predicates come with an 'excluded-middle' ('all-or-nothing') presupposition. The second theory is much closer to the Exclusion account, in that it also relies on an Exh operator appearing in positive sentences to strengthen weak lexical meaning (Bar-Lev 2018, 2021); but on this alternative theory, the alternatives Exh takes are of a different nature, and Exh does not do the same thing with them.

5.1 *The EMP theory*

A classic way to derive the homogeneity effect is to postulate an 'excluded-middle presupposition' (EMP), which ensures that predication only results in the assignment of a truth-value if the predicate holds of all or no parts of its argument (Löbner 2000; cf. Schwarzschild 1994, Gajewski 2005). In what follows, I follow Gajewski's (2005) formalization of this approach for pluralities, and suggest two different ways to carry it over to subatomic homogeneity—neither of which can capture consistent co-predications.

5.1.1 *The EMP with pluralities* Löbner (2000) hypothesizes that predication introduces a presupposition that the predicate holds of all or no parts of its argument. This is the case for both atomic parts of pluralities and subatomic parts of atoms. The EMP straightforwardly results in truth-value gaps for non-homogeneous cases.

An influential formalization of this idea for plural homogeneity comes from Gajewski (2005). He proposes that the EMP is introduced by an obligatory distributivity operator. DIST is defined in (102), where $\sqsubseteq_{AT}$ refers to atomic parthood; (103) provides a sample LF.

(102) $[\![\text{DIST}]\!] = \lambda P.\, \lambda x : \forall y[y \sqsubseteq_{AT} x \rightarrow P(y)] \vee \forall y[y \sqsubseteq_{AT} x \rightarrow \neg P(y)].\, \forall y[y \sqsubseteq_{AT} x \rightarrow P(y)].$

(103) [The children] [DIST [sang]].

In the positive case, DIST asserts that the predicate holds of all atoms in the plurality. This is consistent with the first disjunct in the presupposition. Given the assertion, the only noticeable effect of the presupposition is that, if only some of the children sang, the sentence would be undefined rather than false.

(104) The children sang.

a. PRESUPPOSITION: either all of the children sang or none of the children sang
b. ASSERTION: all the children sang
→ all of the children sang

In the negative case, even if the assertion is only that not all the children sang, the presupposition projects past negation and effectively strengthens the assertion to mean that none of them sang. The assertion is incompatible with the first presuppositional disjunct, so the only remaining possibility is for the second disjunct to hold—that is, for none of the children to have sung.

(105) The children didn't sing.

a. PRESUPPOSITION: either all of the children sang or none of the children sang
b. ASSERTION: not all the children sang
→ none of the children sang

Given the all-or-nothing presupposition, it does not matter that the output condition of DIST is written with ∀ rather than ∃. Having an existential output condition would yield identical definedness and truth conditions, because DIST is only defined for predicates that hold of all or none of their plural argument's atomic parts.

The EMP account of homogeneity has come under various types of criticism in the literature (Spector 2013; Križ 2015), both in terms of the core proposal that a presupposition is at work in creating the homogeneity paradigm, and in terms of Gajewski's linking of this presupposition to distributivity—Križ (2015) shows that homogeneity is also observable with non-distributive plural predication. For the sake of argument, I put these criticisms aside, to focus exclusively on whether the account can capture the quantificational force of summative predicates.

5.1.2 *Consistent co-predications as a problem for the EMP account* For Löbner (2000), subatomic homogeneity results from the same EMP as with pluralities. There are two ways to do this: either the presupposition is present in the lexical entry of summative predicates, or there is a subatomic $\text{DIST}_{\text{SUBAT}}$ operator in addition to the atomic DIST operator posited by Gajewski (2005).

The first option would mean that summative predicates are lexically defined for arguments only if they are true of all or none of their subatomic parts:

(106) $[\![\text{green}]\!] =$
$\lambda x : \forall y[y \sqsubseteq x \rightarrow \mathsf{green}(y)] \vee \forall y[y \sqsubseteq x \rightarrow \neg\mathsf{green}(y)].\ \forall y[y \sqsubseteq x \rightarrow \mathsf{green}(y)].$

Postulating that the EMP is part of *green*'s lexical meaning could be motivated by the fact that the presence of this presupposition is regulated lexically, in light of the summative–integrative distinction. But to deal with Boolean conjunctions of colour terms (107), we need colour terms to have existential lexical meanings.

(107) The flag is green and white.

This is not compatible with the EMP. Writing out the assertive component of (106) as existential changes nothing, as described above for plural homogeneity.

Perhaps this problem could be solved if the EMP was contributed by an operator scoping *above* colour terms, with these being lexically existential. In conjunctions, such an operator could scope above both colour terms at once, as Exh does on the Exclusion account.[20] To see if this works, we first define a subatomic $\text{DIST}_{\text{SUBAT}}$ operator, identical to Gajewski's (102) but with reference to subatomic parts:

(108) $[\![\text{DIST}_{\text{SUBAT}}]\!] =$
$\lambda P.\ \lambda x : \forall y[y \sqsubseteq_{\text{SUBAT}} x \rightarrow P(y)] \vee \forall y[y \sqsubseteq_{\text{SUBAT}} x \rightarrow \neg P(y)].\ \forall y[y \sqsubseteq_{\text{SUBAT}} x \rightarrow P(y)].$

Moving the EMP from colour terms' lexical meaning to an operator creates perfectly acceptable results for non–co-predicational sentences. We will need colour terms to be lexically existential for the conjunction data, so let us assume this right away:

(109) $[\![\text{green}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{green}(y)] \equiv \lambda x.\mathsf{green}_{\exists}(x).$

20 The co-predications made consistent via additives would still need to be explained, so this is not obviously a promising solution.

We get the following meaning once *green* and $\textsc{dist}_{\textsc{subat}}$ compose:[21]

(110) $[\![\textsc{dist}_{\textsc{subat}}\ \text{green}]\!]$
$= \lambda x : \forall y[y \sqsubseteq_{\textsc{subat}} x \rightarrow \mathsf{green}_{\exists}(y)] \vee \forall y[y \sqsubseteq_{\textsc{subat}} x \rightarrow \neg\mathsf{green}_{\exists}(y)].\ \mathsf{green}_{\exists}(x).$

(110) is a good result as far as non–co-predicational sentences are concerned. Despite the existential lexical entry for *green*, the positive disjunct means that all pieces are *all* green: if there was a piece which was not entirely green, the non-green piece of that piece would itself lack a green piece, contrary to the meaning of this disjunct. As for the negative disjunct, this straightforwardly means that there are no pieces with any green on them.

Let us now attempt to obtain consistent conjunctions by having $\textsc{dist}_{\textsc{subat}}$ scope over both colour terms at once:

(111) The flag is [$\textsc{dist}_{\textsc{subat}}$ [green and white]].

Before the functional application of $\textsc{dist}_{\textsc{subat}}$, the meaning of the conjunction is:

(112) $[\![\text{green and white}]\!] = \lambda x.\ \mathsf{green}_{\exists}(x) \wedge \mathsf{white}_{\exists}(x).$

$\textsc{dist}_{\textsc{subat}}$ then takes the entire conjunction as its argument, producing (113).

(113) $[\![\textsc{dist}_{\textsc{subat}}\ [\text{green and white}]]\!] =$
$\lambda x : \forall y[y \sqsubseteq_{\textsc{subat}} x \rightarrow [\mathsf{green}_{\exists}(y) \wedge \mathsf{white}_{\exists}(y)]]$
$\vee \forall y[y \sqsubseteq_{\textsc{subat}} x \rightarrow \neg[\mathsf{green}_{\exists}(y) \wedge \mathsf{white}_{\exists}(y)]].$
$\mathsf{green}_{\exists}(x) \wedge \mathsf{white}_{\exists}(x).$

This is not a welcome result. The first disjunct requires all pieces of the flag to themselves have both a green and a white piece. As such, however small a piece you choose, it would have to be made up of a white piece and a green piece, and these white pieces and green pieces are themselves divisible between white pieces and green pieces, and so on infinitely. We would have needed this disjunct of the presupposition to contain a disjunction ('all parts are $\text{green}_{\exists}$ *or* $\text{white}_{\exists}$') rather than a conjunction. We could try to turn to the second disjunct to see if this somehow gets the right result for positive sentences, but it does not: a white and green flag can have a piece that has both white and green on it.

The conclusion is that there is at least one co-predication, namely conjunction, that the EMP theory of homogeneity cannot capture.

5.2 *The Inclusion theory*

The second account of homogeneity we consider in this section is Bar-Lev's (2018, 2021); it is also based on exhaustivity, giving it some common ground with the Exclusion theory (see also Magri 2014). The basic premise is that the lexical meaning of plurals is existential, immediately capturing the truth conditions of negative sentences. In positive sentences, Exh strengthens the existential to a universal. Bar-Lev uses the notion of Innocent Inclusion

21 As proposed by Gajewski (2005), the purpose of the EMP is mainly to strengthen the meaning of negative sentences; for positive sentences, its only effect is to create heterogeneity-gaps. The proposal embodied by (110) effectively uses the EMP for the opposite of this: the EMP would strengthen positive sentences, and have no effect on negative ones other than creating heterogeneity-gaps.

(Bar-Lev and Fox 2017; Bar-Lev 2018) to have Exh assert the truth of ('include') the domain alternatives of the existential plural. I will focus exclusively on the theory Bar-Lev builds for distributive plural homogeneity, which is all that is needed to try to carry over his account to summative predicates.

5.2.1 *Bar-Lev's theory for plural homogeneity* In Bar-Lev's theory, the meaning of plurals is existential: prior to exhaustification, the meaning of *the children laughed* is that at least one laughed. In what follows, assume there are two children, Adam and Jade.

(114) $[\![\text{The kids laughed}]\!] = 1$ iff $\mathsf{laughed}(a) \vee \mathsf{laughed}(j)$.

This existential plain meaning comes about from an existential plural operator, ∃-PL, which takes as its first argument a domain variable ((115)–(117) are from Bar-Lev 2021:1062):

(115) a. $[\![\exists\text{-PL}]\!] = \lambda D.\lambda P.\lambda x.\exists y \in D \cap Part_{\text{AT}}(x)[P(y) = 1]$.
b. $Part_{\text{AT}}(x) = \{y : y \sqsubseteq_{\text{AT}} x\}$

(114) therefore has the LF in (116), where the domain D is presented as a subscript on ∃-PL.

(116) [The kids] [$\exists\text{-PL}_D$ laughed].

(116) obtains the meaning in (117), which (assuming that $[\![\text{the kids}]\!] = a \oplus j$ and $D = \{a, j\}$) is equivalent to (114).

(117) $[\![(116)]\!] = 1$ iff $\exists y \in D \cap Part_{\text{AT}}([\![\text{the kids}]\!])[\mathsf{laughed}(y) = 1]$.

Naturally, (117) immediately obtains the intended meaning for negative sentences, which mean that there is no individual that (*i*) is in the domain, (*ii*) is part of the denotation of *the kids*, and (*iii*) laughed. Thus, we get the intended meaning that no child laughed. In the positive, the sentence must be strengthened; for Bar-Lev (2018, 2021), the alternatives for Exh are obtained by replacing the sentence's domain with subdomains:

(118) ALT = {Adam laughed ∨ Jade laughed, Adam laughed, Jade laughed}

The subdomain alternatives 'Adam laughed' and 'Jade laughed' are not innocently excludable (Fox 2007). Excluding them would dysfunctionally result in the sentence meaning that Adam or Jade laughed, but neither Adam nor Jade laughed. Innocent Exclusion is defined as in (119):

(119) **Innocent Exlusion procedure:** (Bar-Lev 2021:1066)
a. Take all maximal sets of alternatives that can be assigned false consistently with the prejacent.
b. Only exclude (i.e., assign false to) those alternatives that are members in all such sets—the **Innocently Excludable** alternatives.

Further, the set of alternatives (118) is not closed under conjunction: there is no strong alternative of the form 'Adam laughed *and* Jade laughed' for Exh to exclude.

From here, Bar-Lev relies on the notion of Innocent Inclusion. This is the idea that Exh *includes* all alternatives that are not excluded and which can be included consistently:

(120) **Innocent Inclusion procedure:** (Bar-Lev 2021:1067)

a. Take all maximal sets of alternatives that can be assigned true consistently with the prejacent and the falsity of all [innocently excluded] alternatives.
b. Only include (i.e., assign true to) those alternatives that are members in all such sets—the **Innocently Includable** alternatives.

Thus, Exh asserts the alternatives 'Adam sang' and 'Jade laughed.' This results in the meaning that all the children laughed.

5.2.2 *Carrying Bar-Lev's theory over to subatomic homogeneity: a first attempt* Let us start by seeing how Bar-Lev's theory for plural homogeneity can be carried over to non–co-predicated summative predicates. First, we need existential meaning for colour terms. (121) would be the familiar way to do this, but it lacks the domain variable that Bar-Lev relies on to create subdomain alternatives.

(121) $[\![\text{green}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{green}(y)]$.

Instead, let us imitate Bar-Lev's ∃-PL in postulating an operator taking a domain variable—call it ∃-SG:

(122) a. $[\![\exists\text{-SG}]\!] = \lambda D.\lambda P.\lambda x.\exists y \in D \cap Part(x)[P(y) = 1]$.
b. $Part(x) = \{y : y \sqsubset x\}$

D in (122) must be populated not just by individuals (atoms) and pluralities, but also subatomic pieces. For presentation, let us assume there are two subatomic pieces to the flag, A and B. But keep in mind that, unlike the toy model used above for *the kids*, this toy model is truly misleading, since unlike pluralities, atoms are divisible between an infinite number of arbitrary, overlapping pieces.

From here, the idea for a sentence like (123a) is that *green* is an argument of the ∃-SG operator, whose domain argument creates subdomain alternatives. Prior to the merger of Exh, (123a) has the LF in (123b).

(123) a. The flag is green.
b. $[_{\nu P}\ [_{DP}$ The flag$]\ [_{\nu P}$ is $\exists\text{-SG}_D$ green$]]$

Still following Bar-Lev, (123b) is exhaustified in positive sentences (124a) but not negative ones (124b).

(124) a. $[_{\nu P}\ \text{Exh}_{\text{ALT}}\ [_{\nu P}\ [_{DP}$ the flag$]\ [_{\nu P}$ is $\exists\text{-SG}_D$ green$]]]$
b. $[_{\nu P}$ not $[_{\nu P}\ [_{DP}$ the flag$]\ [_{\nu P}$ is $\exists\text{-SG}_D$ green$]]]$

As for the meaning of colour terms, since we will need them to be existential for the conjunction data, let us only consider that possibility (but see section 5.2.3). If *green* is existential, the meaning we get from (124) for negative sentences is that there is no piece with any green on it, as desired.

(125) $[\![(124\text{b})]\!] = 1$ iff $\neg\exists y \in D \cap Part(\iota x[\mathsf{flag}(x)])[\mathsf{green}_{\exists}(y)]$.

Prior to exhaustification, the meaning obtained in the positive is that there is a piece of the flag which is partly green:

(126) $[\![(123b)]\!] = 1$ iff $\exists y \in D \cap Part(\iota x[\mathsf{flag}(x)])[\mathsf{green}_\exists(y)]$.

In our toy model where the pieces are A and B, (126) is equivalent to (127):

(127) $\mathsf{green}_\exists(a) \vee \mathsf{green}_\exists(b)$.

The alternatives triggered by the *D* variable in (124a) are the following:

(128) $\text{ALT} = \left\{ \begin{matrix} \text{A is green}_\exists \vee \text{B is green}_\exists, \\ \text{A is green}_\exists, \\ \text{B is green}_\exists \end{matrix} \right\}$

Of course, the set of alternatives is actually infinite because the subject (the flag) can be cut up in an infinite amount of (possibly overlapping) pieces of arbitrary sizes. Following the Innocent Inclusion of all the alternatives in (128), we obtain the meaning that all pieces are partly green. Given that pieces can be subdivided into further pieces, this means that all pieces are in fact entirely green.

However, conjoined summative predicates pose the same problem for this theory as they did for the EMP. Given our toy model, the pre-exhaustification meaning of (129a) is in (129b) ('some part of the flag is both white and green').

(129) a. The flag is white and green.
b. $[\![(129a)]\!] = 1$ iff (A is white$_\exists$ $\wedge$ A is green$_\exists$) $\vee$ (B is white$_\exists$ $\wedge$ B is green$_\exists$)

The alternatives are obtained by replacing the domain with subdomains:

(130) $\text{ALT} = \left\{ \begin{matrix} \text{A is white and green} \vee \text{B is white and green}, \\ \text{A is white and green}, \\ \text{B is white and green} \end{matrix} \right\}$

Now we exhaustify (129). No alternatives are excludable. Whether any are includable depends on how much Exh takes world knowledge about colours and surfaces into account. If we include all the alternatives, the meaning we end up with is that A is partly white and partly green, and B is also partly white and partly green. In other words, all pieces of the flag are themselves divisible between a partly green piece and a partly white piece, which are themselves divisible between white and green pieces, and so on—the same problem we had with the EMP. If we tried to get around this problem by claiming that the alternatives are not innocently includable, this would create another problem: no strengthening would take place at all, and the sentence would only mean that some piece of the flag is partly white and partly green.

5.2.3 *A second attempt at using the Inclusion theory for summative predicates* So far, it does not appear that the Inclusion theory can capture co-predications of summative predicates. However, a *Journal of Semantics* reviewer suggests a way that Bar-Lev's (2021) Inclusion theory as applied to summative predicates might work for conjunctions (131) after all.

(131) The flag is white and green.

My discussion in section 5.2.2 did not consider using *or* as an alternative to *and*—doing so makes it possible to capture (131). In this subsection, I first explain how this is so, then point out that the theory makes one of the same incorrect predictions as Krifka's (1990) claim outlined in section 3.2.2 that *and* in (131) is non-Boolean.

Let us assume that colour terms are universal, and that Exh knows that they therefore cannot both hold of a particular part of a surface.[22] Conjunctions come with an ∃-SG operator on each conjunct:

(132) Exh_{ALT} [the flag is [$\exists\text{-SG}_D$ white] and [$\exists\text{-SG}_D$ green]].

Prior to exhaustification, the basic meaning of this sentence is that there is a part of the flag which is entirely white, and there is a part which is entirely green.

(133) $[\![$The flag is [$\exists\text{-SG}_D$ white] and [$\exists\text{-SG}_D$ green]$]\!] = 1$ iff
$\exists y \in D \cap Part(\iota x[\mathsf{flag}(x)])[\mathsf{green}_\forall(y)] \wedge \exists y \in D \cap Part(\iota x[\mathsf{flag}(x)])[\mathsf{white}_\forall(y)]$.

If the two parts of the flag are A and B, this is equivalent to:

(134) $(\mathsf{white}_\forall(a) \vee \mathsf{white}_\forall(b)) \wedge (\mathsf{green}_\forall(a) \vee \mathsf{green}_\forall(b))$

Since it does not have to be the same part that is white/green, the meaning is consistent. But the result so far is only that each colour term is existential; we must still strengthen them to being existential-plus.

The alternatives for Exh are obtained both by replacing D with subdomains, and replacing $P \wedge Q$ with the normal alternatives for conjunction: P, Q, and $P \vee Q$. That is, in addition to domain-restriction, (134) has the following alternatives:

(135) $\left\{\begin{array}{c}\text{The flag is white and green,}\\ \text{The flag is white or green,}\\ \text{The flag is white,}\\ \text{The flag is green}\end{array}\right\}$

The alternatives in (135) multiply once we also take domain-restriction into account (136).[23] I assume again for convenience that the flag has two parts, A and B. To make it clear where the alternatives come from, there is a guide on the right-hand side of (136), referring both to the size of the domain and whether the meaning comes from *and*, *or*, or a left/right conjunct.

(136) a. $(\mathsf{white}_\forall(a) \vee \mathsf{white}_\forall(b)) \wedge (\mathsf{green}_\forall(a) \vee \mathsf{green}_\forall(b))$ $D = \{a, b\}$; 'and'
b. $\mathsf{white}_\forall(a) \wedge \mathsf{green}_\forall(a)$ $D = \{a\}$; 'and'

22 This is necessary because we will need Exh never to assert that two colours hold of the same part of a surface. The reviewer suggests this could arise because colour terms are mutually exclusive as a matter of logic (perhaps the lexical meaning of *green* is '*x* is green and not white and not red and not . . .'). However, this would almost certainly be impossible to square with the co-predications made consistent through an additive. Additives cannot make mutually incompatible predicates compatible (see Paillé 2022b:ch. 2). As such, something else would need to let Exh know that two colour terms cannot both hold of the same part of a surface. It is not clear what this would be, given Magri's (2009) argument that Exh does not take world knowledge into account (cf. section 4.2.1 and in particular fn. 10, where I argue that Exh can see the entailment relations between colour predicates; this is different from claiming that it knows that a surface can only have one colour).

23 I will assume for simplicity that the domains of the two ∃-SG operators co-vary—nothing will hinge on this.

c. $\text{white}_\forall(b) \wedge \text{green}_\forall(b)$ $D = \{b\}$; 'and'
d. $(\text{white}_\forall(a) \vee \text{white}_\forall(b)) \vee (\text{green}_\forall(a) \vee \text{green}_\forall(b))$ $D = \{a, b\}$; 'or'
e. $\text{white}_\forall(a) \vee \text{green}_\forall(a)$ $D = \{a\}$; 'or'
f. $\text{white}_\forall(b) \vee \text{green}_\forall(b)$ $D = \{b\}$; 'or'
g. $\text{white}_\forall(a) \vee \text{white}_\forall(b)$ $D = \{a, b\}$; left conjunct
h. $\text{green}_\forall(a) \vee \text{green}_\forall(b)$ $D = \{a, b\}$; right conjunct
i. $\text{white}_\forall(a)$ $D = \{a\}$; left conjunct
j. $\text{white}_\forall(b)$ $D = \{b\}$; left conjunct
k. $\text{green}_\forall(a)$ $D = \{a\}$; right conjunct
l. $\text{green}_\forall(b)$ $D = \{b\}$; right conjunct

(136a) is the basic meaning of the assertion. As for the other alternatives, which ones are excluded or included? Only the internally inconsistent (136b) and (136c) can be excluded. (136i)–(136l) are neither excluded nor included; including them together would result in inconsistency. As for (136d)–(136h), these cannot be excluded, but they can be included. Thus, the meaning of (132) is (137):

(137) $(\text{white}_\forall(a) \vee \text{white}_\forall(b)) \wedge (\text{green}_\forall(a) \vee \text{green}_\forall(b))$
$\wedge(\text{white}_\forall(a) \vee \text{green}_\forall(a)) \wedge (\text{white}_\forall(b) \vee \text{green}_\forall(b))$

The first line in (137) entails that some part is white and some part is green; the second line entails that all parts are either white or green.[24] We have obtained a consistent conjunction with existential-plus meaning for the colour terms.[25]

However, notice that we have just created meaning similar to what is obtained with a non-Boolean *and*, as discussed in section 3.2.2. In that section, I criticized the view that *and* in colour conjunctions (with an atomic argument) is non-Boolean in part due to the following datapoint:

(138) #The flag is completely white and completely green.

We saw that Krifka (1990) predicts (138) to be acceptable due to his suggestion that colour terms are lexically universal, with *and* breaking up the flag in two. But something similar goes with the Inclusion theory we have just seen. The colour terms are universal, so nothing should go wrong if we replaced them with explicitly universal expressions like *completely white*. I assume that *completely* is a universal:

(139) $[\![\text{completely}]\!] = \lambda P.\lambda x.\forall y[y \sqsubseteq x \rightarrow P(y)]$.

24 In our toy model, there are only two parts, A and B. Because of this, the first line in (137) would be enough to mean that all of the flag is white or green and the flag has both white and green. But in a model where the flag has three parts (A, B, and C), the first line in (137) (augmented with the disjuncts '$\text{white}_\forall(c)$' and '$\text{green}_\forall(c)$') is no longer enough: it would only entail that at least two of the three parts of the flag are white or green. This is why the second line in (137) is also necessary, since it carries the entailments that for each part, that part is white or green.

25 In fact, (137) is only consistent if the parts of the flag are non-overlapping. Stepping outside of our toy model, the pieces of atoms are infinite and possibly overlapping. As such, this theory would require the flag to be partitioned first to ensure non-overlap of pieces.

Therefore, if colour terms are lexically universal, *completely white* and *white* have the same meaning, and the truth conditions for (138) (on this Inclusion theory) are wrongly predicted to be identical to the basic conjunction:

(140) The flag is white and green.

I end this subsection by pointing out that, even if it worked for conjunction, this alternative approach does not carry over to the co-predications with *also* (141). *Also*, unlike *and*, does not have a disjunctive alternative.

(141) The white flag is also green.

One could try to claim that *also* is capable of pruning domain-alternatives. Assuming again that there are two parts, A and B, (141) would mean that part A of the flag is white (the B alternative from '∃-SG_D white' being pruned), while part B is green (the A alternative being pruned from '∃-SG_D green'). But this would in effect end up creating a 'non-Boolean *also*', which is not in fact intuited. For instance, this would predict that *also* should also be able to make plural predication with inconsistent predicates consistent, which is not the case:

(142) #The 3-billion-year-old planets are also 5 billion years old.

Assume there are four planets, A–D. If *also* could prune domain-alternatives as needed for (141), (142) would be expected to have truth conditions compatible with planets A and B (but not C or D) being 3 billion years old, and planets C and D (but not A and B) being 5 billion years old. The additive could prune the alternatives corresponding to planets C and D from '∃-PL_D 3-billion-year old', and the alternatives corresponding to planets A and B from '∃-PL_D 5 billion years old'. Since this apparently cannot occur in (142), such pruning presumably also cannot occur in (141).

5.3 *Section summary*

In this section, I have overviewed two theories of homogeneity, which both posit semantically universal quantification in positive sentences: the EMP and Inclusion theories. While both can predict the quantificational force of colour terms in non–co-predicational sentences, they fall on the same problem for consistent co-predications: they create the meaning that each conjunct must be true of all arbitrary pieces of the subject. At least, this is the case on my initial attempt at using the Inclusion theory for summative predicates; the problem for the Inclusion theory is different if *or* is an alternative to *and*. Thus, even if the Exclusion theory of summative predicates cannot be extended to plural predication, it is empirically preferable for summative predicates to the two alternatives seen so far.

The Inclusion and Exclusion accounts share an important component: they both involve weak lexical meaning together with covert exhaustification in positive sentences, but not in negative sentences.[26] Moreover, as hinted at in section 4 and discussed in section 7, it is not only the Exclusion theory (for subatomic homogeneity) that must posit a locality condition on exhaustification; the Inclusion theory (for plural homogeneity) must do so too. The fact that the Exclusion account cannot be extended to plural homogeneity does

26 Given that I adopted Pexh in section 4, it would be more accurate to say that their exhaustification in negative sentences is vacuous for the truth-conditions, while still leading to a truth-value gap.

not make it incorrect for subatomic homogeneity, and vice-versa for the Inclusion account. Given how much they have in common, each theory can be taken to correctly explain one side of the paradigm. If so, homogeneity (subatomic and plural) is a local and obligatory exhaustification effect whereby the weak lexical meaning of certain quantificational elements (summative predicates and the plural operator) is strengthened in positive sentences; but this strengthening occurs in different ways for summative predicates and the plural operator, due to the different nature of the alternatives.

6. THEORIES OF HOMOGENEITY THAT CREATE TOO FEW CONTRADICTIONS

In this section, we turn to a second set of theories on homogeneity, in particular work suggesting that the homogeneity paradigm involves underspecification between existential and universal quantificational force, with some mechanism determining which is actually intuited in a given sentence. Such theories cannot explain the co-predicational paradigm, but for different reasons from the EMP and Inclusion accounts. One of them (a pragmatic account based on the Strongest Meaning Hypothesis) clearly creates too few contradictions; inconsistent co-predications are predicted to be consistent. The other, a theory based around the co-assertion of 'candidate interpretations', does not (as currently stated) make clear predictions, but it either predicts inconsistent co-predications to be consistent or vice-versa. I present it in this section because its basic insight essentially builds on the first underspecification theory.

6.1 *The Pragmatic Underspecification theory*

Recall the classical homogeneity paradigm:

(143) a. Adam saw the kids.
$\approx$ he saw all of the kids
$\not\approx$ he saw at least some of the kids
b. Adam didn't see the kids.
$\not\approx$ he did not see all of the kids
$\approx$ he saw none of the kids

One theory about this builds on the simple intuition that each sentence in (143) has the quantificational force that provides the strongest global meaning. Krifka (1996), Lasersohn (1999), Winter (2001), and Malamud (2012) propose that grammar does not fix whether a predicate is interpreted universally or existentially when it takes a plurality as an argument. Rather, a pragmatic principle ensures that speakers prefer whichever interpretation results in the strongest meaning:

(144) If a predicate P applies to a sum individual x, grammar does not fix whether the predication is universal ($\forall y[y \sqsubseteq x \rightarrow P(y)]$) or rather existential ($\exists y[y \sqsubseteq x \wedge P(y)]$), except if there is explicit information that enforces one or the other interpretations. (Krifka 1996:146)

This is inspired by the Strongest Meaning Hypothesis (SMH) that Dalrymple *et al.* (1994) propose for reciprocals.

We might attempt to carry over the Pragmatic Underspecification theory of homogeneity to the subatomic case. On this view, *green* is lexically ambiguous (145), and whichever meaning is strongest in a particular sentence will be the one chosen.[27]

(145) $[\![\text{green}]\!] =$

a. $\lambda x.\exists y[y \sqsubseteq x \wedge \textsf{green}(y)]$.

b. $\lambda x.\forall y[y \sqsubseteq x \rightarrow \textsf{green}(y)]$.

On this pragmatic theory of homogeneity, computing the strength of predicates should never lead to inconsistency. After all, the quantificational strength of predicates results from a pragmatic preference for strong meanings over weaker ones; this preference would be overridden if the strongest meaning was inconsistent. Clearly, this is impossible to maintain in light of the inconsistent co-predications:

(146) a. #The white flag is green.

b. #The white green flag is high.

Such examples straightforwardly counter the prediction of the Pragmatic Underspecification account; the pragmatics would not create inconsistency out of potentially consistent lexical material.[28]

One could try to defend this pragmatic theory by claiming that the pragmatic principle/SMH is blind to world knowledge; it would function like Magri (2009) claims that Exh functions, looking at logical entailment alone. Since there is no *logical* contradiction in a flag being entirely white and entirely green, the SMH would always apply to colour terms even if this creates inconsistency. While this would let the SMH capture the inconsistent half of the co-predication paradigm, we would lose the consistent half: summative predicates co-predicated via *and* or *also* would now be expected to involve universal colour terms.

27 This is a slight oversimplification. For the consistent co-predications, recall we will need an 'existential-plus' meaning, which neither of the possibilities in (145) can capture. Something similar exists for reciprocals, which can also be stronger than existential while still not being universal. Indeed, (i) does not mean that each player sat alongside each player (which is impossible, given that humans have two sides); nor does it mean that each player sat alongside at least one player (in which case the players could have sat in two entirely separate groups).

(i) Five Boston pitchers sat alongside each other. (Dalrymple *et al.* 1994:73)

Rather, (i) is 'existential-plus' in some sense. It may be possible to capture the existential-plus meaning of colour terms in consistent co-predications in the same way. Either way, I will be arguing against the Underspecification theory on independent grounds due to *inconsistent* co-predications, so this is ultimately orthogonal.

28 A rather similar problem might hold for the discussion of colour terms by Kennedy and McNally (2010). Their description differs from mine in not taking colour terms to be universal in positive sentences in the basic case. They treat colour terms as vague degree predicates, taking a positive degree argument *pos*. Since *pos* can in principle be of any value, this theory does not predict contradictions ever to be intuited (at least sentence-internally); the listener would choose a *pos* value which leads to consistency. One could attempt to put various sorts of constraints on what the value of *pos* can be, in order to derive contradictions, but it is not clear what the nature of these constraints would be.

6.2 *The Co-Assertion theory*

Križ and Spector (2021) also provide an underspecification theory, but not based in the SMH. They point out that the SMH cannot capture the meaning of pluralities in non-monotonic contexts:

(147) Exactly one student read the books. (Križ and Spector 2021:1135)

Instead, Križ and Spector (2021) suggest that the possible meanings of plurals arise from the *conjunction* of candidate interpretations (CIs). The meaning of (147) is predicted if the two CIs in (148) are conjoined.

(148) a. Exactly one student read some of the books.
b. Exactly one student read all of the books.

Call this the Co-Assertion account of homogeneity. The CIs posited by this theory quickly become far more complex than those in (148), in part because Križ and Spector (2021) aim to account for plural homogeneity with both distributive and collective predication; this is not a complication we need to linger on here. In a nutshell, Križ and Spector (2021:§3.1) suggest that, rather than those in (148), CIs are disjunctions in which the predicate is predicated of various parts of the plurality. Consider (149a). For each individual y that is part of a plurality x, there is a CI where one disjunct is of the form $P(y)$, and for each sum z that contains y, there is also a disjunct of the form $P(z)$. There are also CIs corresponding to the disjunction of those CIs. Hence, if there are two children, A and B, the CIs for (149a) are in (149b) (Križ and Spector 2021:1160).

(149) a. The children sang.
b. $$\text{CIs} = \left\{ \begin{matrix} a \text{ sang} \vee b \text{ sang} \vee a \oplus b \text{ sang}, \\ a \text{ sang} \vee a \oplus b \text{ sang}, \\ b \text{ sang} \vee a \oplus b \text{ sang}, \\ a \oplus b \text{ sang} \end{matrix} \right\}$$

For distributive predication, (149b) is equivalent to:

(150) $$\text{CIs} = \left\{ \begin{matrix} a \text{ sang} \vee b \text{ sang}, \\ a \text{ sang}, \\ b \text{ sang}, \\ a \oplus b \text{ sang} \end{matrix} \right\}$$

What is more, to capture non-maximality, Križ and Spector (2021) posit that not all CIs actually end up being co-asserted; this depends on the QUD. It is only the CIs that are 'strongly relevant' (they are 'strongly relevant candidate interpretations', SRCIs) to a QUD which are actually co-asserted. Assume (cf. section 2) that worlds are partitioned according to how they resolve a QUD. SRCIs are CIs that correspond exactly to a cell or a set of cells in a partition of worlds:

(151) A proposition p is STRONGLY RELEVANT to a partition [of worlds] I iff $\exists X \subset I$: $p = \bigcup X$. (Križ and Spector 2021:1145)

That is, p is strongly relevant iff there is a cell (corresponding to a single cell or a set of cells) in the partition of worlds such that p denotes all the worlds in that cell, and no other. What matters for us is that, if the QUD is *Who sang?*, for each atomic or plural individual who is a child, there will be a cell in the partition corresponding to that child singing. In fact, all the CIs in (149b) are SRCIs on this QUD.[29] Conjoining them yields the meaning that every child sang.

Turning now to subatomic homogeneity, let us consider the sentence (152) in the context of a QUD making each part of the flag relevant (such as 'What colour is the flag?' or 'What does the flag look like?'—corresponding to the QUDs one accommodates upon hearing (152) out of the blue).

(152) The flag is green.

For simplicity of presentation, take the flag to have two parts, A and B—although as with my toy model used for the Inclusion theory (section 5.2), we have to keep in mind that this is quite misleading since we are dealing with subatomic pieces. The SRCIs are the following:

(153) $$\text{SRCIs} = \left\{\begin{array}{c} a \text{ is green} \vee b \text{ is green} \vee a \oplus b \text{ is green}, \\ a \text{ is green} \vee a \oplus b \text{ is green}, \\ b \text{ is green} \vee a \oplus b \text{ is green}, \\ a \oplus b \text{ is green} \end{array}\right\}$$

This is equivalent to:

(154) $$\text{SRCIs} = \left\{\begin{array}{c} a \text{ is green} \vee b \text{ is green}, \\ a \text{ is green}, \\ b \text{ is green}, \\ a \oplus b \text{ is green} \end{array}\right\}$$

For (152), conjoining the SRCIs in (153)/(154) creates the meaning that the entire flag is green (regardless of whether *green* is lexically existential or universal).

The theory cannot deal with the co-predication paradigm, however. In particular, it predicts the consistent and inconsistent halves of the paradigm to behave the same way. Due to a question about the nature of the co-assertion mechanism left open by Križ and Spector (2021), it is not clear whether both are predicted to be consistent, or both inconsistent. Let us focus on the following subset of the co-predication paradigm:

(155) a. This is a white and green flag. (conjoined adjectives; consistent)
b. #This is a white green flag. (stacked adjectives; inconsistent)

We begin with the inconsistent (155b); if we make the assumptions necessary for the Co-Assertion theory to capture (155b), we will then be unable to explain (155a).

(155b) cannot be explained through a single SRCI, since there is no world in which a flag is all white and all green. On the other hand, it would be possible to obtain (155b) by having a set of SRCIs which are internally consistent, but which result in inconsistency when co-asserted. We would need SRCIs as in (156) (simplifying somewhat in irrelevant ways, e.g.

29 This may make it obscure to the reader why the notion of SRCIs is necessary. Consider if the QUD was 'Did any child sing?'. On this QUD, the only SRCI is the existential 'a sang $\vee$ b sang $\vee$ $a \oplus b$ sang'. Co-asserting this CI with nothing but itself yields an existential meaning for the sentence (149a).

there is nothing about flags in (156)), where the colour terms must be existential for each SRCI to be internally consistent:

(156) SRCIs for (155b) = $\left\{\begin{array}{c} \dots \\ a \text{ is white}_\exists \wedge a \text{ is green}_\exists, \\ b \text{ is white}_\exists \wedge b \text{ is green}_\exists, \\ \dots \end{array}\right\}$

By conjoining the SRCIs, we obtain the impossible flag already discussed for the EMP and Inclusion theories of homogeneity, where each piece must somehow have a piece of each colour. Recall once again that it is only in my presentation that there are two subatomic pieces A and B—in reality, *white and green* would be predicated of every one of the infinity of possibly overlapping subatomic parts making up the flag.

This obtains (155b). But crucially, for this to occur, the mechanism whereby SRCIs are co-asserted must be blind to the creation of inconsistency. There is no equivalent of Bar-Lev and Fox's (2017) 'Innocent Inclusion' in co-assertion—no Innocent Co-Assertion, which would have barred the SRCIs in (156) from being co-asserted. Yet, claiming there is no Innocent Co-Assertion wrongly predicts (155a) to be inconsistent, too. The SRCIs for (155a) look exactly as in (156) (the logical conjunction surfaces for (155b) due to Predicate Modification rather than the lexical entry for *and*, but they are semantically the same), and inconsistency is predicted just as for (155b). For (155a) to be consistent, we need Innocent Co-Assertion, both bringing back the puzzle of (155b) and failing to capture the existential-plus meaning of the colour terms in (155a), since no strengthening would occur.

6.3 *Summary of sections 5 and 6*

The quantificational force of summative predicates is not aligned with the predictions of underspecification theories of homogeneity, at least in the domain of subatomic homogeneity. The pragmatic approach predicts all co-predicated summative predicates to be weak, contrary to fact; the Co-Assertion approach predicts all co-predicated predicates to pattern together (as consistent or inconsistent, depending on assumptions about co-assertion), rather than making the cut between consistent and inconsistent co-predications.

What have we accomplished in sections 5 and 6? In section 4, I built a theory of the quantificational force of summative predicates that can capture both the homogeneity and co-predicational paradigms. But this Exclusion theory cannot be extended to plural predication. I therefore attempted to find a theory of plural homogeneity that can be translated to subatomic homogeneity. We have just gone through four different theories to see that none of them are able to handle the co-predication paradigm, making the Exclusion account the only currently existing option. This exercise has highlighted the importance of the co-predication paradigm in finding the right way to capture homogeneity.

7. CONCLUSION

In this article, I laid out an empirical paradigm touching on the quantificational force of summative predicates that provides new ways to evaluate various theories of homogeneity. I developed a new theory that can obtain the facts observed, and showed that the other currently existing theories of homogeneity cannot do so.

The paradigm, a set of CO-PREDICATIONS, involves summative predicates sometimes being interpreted as strictly universal in positive sentences, even if this leads to sentence-internal inconsistency; but at other times being interpreted as existential-plus and consistent:

(157) TWO KINDS OF CO-PREDICATIONS:

a. *Co-predications where summative predicates are universal and inconsistent:*
   (i) #The white flag is green.
   (ii) #The white green flag is at half-mast.
b. *Co-predications where summative predicates are non-universal and consistent:*
   (i) The white flag is also green.
   (ii) The flag is white and green.

I showed that the Presuppositional (Löbner 2000, Gajewski 2005), Inclusion (Bar-Lev 2018, 2021), Pragmatic Underspecification (Krifka 1996 and others), and Co-Assertion (Križ and Spector 2021) theories of homogeneity, as translated from the domain of plural predication to atomic summative predication, all fail to capture this paradigm.

I therefore suggested to update the Exclusion theory of Harnish (1976) and Levinson (1983), according to which the meaning of summative predicates results from the exclusion of related predicates (colour terms exclude other colour terms, material terms other material terms, and so on):

(158) $[\![\text{Exh}_{\text{ALT}} \text{ [the flag is green]}]\!] = 1 \text{ iff } \mathsf{green}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f).$

But I modified this theory in two ways. First, I adopted the Pexh operator of Bassi *et al.* (2021) in order to capture heterogeneity-gaps:

(159) $$[\![\text{Pexh}_{\text{ALT}} \text{ [the flag is green]}]\!] = \begin{cases} 1, \text{ iff } \mathsf{green}_{\exists}(f) \wedge \neg\mathsf{white}_{\exists}(f) \wedge \neg\mathsf{red}_{\exists}(f); \\ 0, \text{ iff } \neg\mathsf{green}_{\exists}(f); \\ \#, \text{ otherwise} \end{cases}$$

Second, I showed that the exhaustivity operator must be subject to a locality constraint in order to capture that summative predicates are exclusive of one another in some co-predicational positive sentences and all non–co-predicational positive sentences (modulo non-maximality), e.g.:

(160) a. $[\![\text{Exh}_{\text{ALT}} \text{ [exactly one flag is green]}]\!]$
   $= 1$ iff exactly one flag is green$_{\exists}$ $\wedge$ not exactly one flag is red$_{\exists}$ $\wedge$ not exactly one flag is white$_{\exists}$
   $\Rightarrow$ not an intuited meaning
b. $[\![\text{Exactly one flag is [Pred-Exh}_{\text{ALT}} \text{ green]}]\!]$
   $= 1$ iff exactly one flag is green$_{\exists}$ and no other colour
   $\Rightarrow$ the intuited meaning

One criticism of exhaustivity accounts of homogeneity (e.g. Križ and Spector 2021), namely that homogeneity does not preferably disappear in downward-entailing contexts, no longer holds as such. It is in fact a *general* fact of homogeneity that it is computed locally, whether the environment is DE or not.

While the Exclusion account does not carry over to plural homogeneity, I suggest this is not problematic. It is reasonable to state that what subatomic and plural homogeneity have in common is that they are both locally computed exhaustivity effects, but with alternatives of a different nature and, therefore, a difference in whether Exh includes or excludes them. Indeed, plural homogeneity behaves like subatomic homogeneity in necessarily being computed locally; for example, it does not preferably disappear in DE environments (see section 4.2.2):

(161) If you solve the problems, you will pass the exam. (Križ 2015:27)
≈ 'If you solve **all** the problems, you will pass the exam.'

If they are both locally computed exhaustification effects, this means that subatomic and plural homogeneity have substantial theoretical common ground, even if they arise in partly different ways.

I conclude by pointing out something important about how the meaning of summative predicates fits among the meanings of predicates generally (Paillé 2022b). The Exclusion theory can do something that the other theories of subatomic homogeneity cannot: it collapses the judgments for co-predications of summative predicates with co-predications of non-summative predicates, even though subatomic quantification is only a factor in the former case. Indeed, the co-predication paradigm carries over to integrative predicates completely, suggesting that summative and integrative predicates are in fact subject to the same Exclusion effect. Compare (157) and (162):[30]

(162) Two kinds of co-predications with integrative predicates:
a. *Co-predications where **integrative** predicates are inconsistent:*
(i) #The comedy is a tragedy.
(ii) (N/A; nouns; see fn. 30)
b. *Co-predications where **integrative** predicates are consistent:*
(i) The comedy is also a tragedy.
(ii) The play is (both) a comedy and a tragedy.

A play is a comedy or tragedy not by virtue of its parts being comedies or tragedies, but by virtue of meeting a set of requirements in its entirety; the predicates are integrative, and one cannot capture the pattern in (162) with a theory about part-structure. Most of the theories of homogeneity discussed in this article, however, are explicitly about part-structure, so that they would need to explain (157) and (162) through entirely different mechanisms. The exception is the Pragmatic Underspecification theory (section 6.1), whose basic insight (that predication is ambiguous and the strongest consistent meaning is chosen) is not exclusively made for part-structure, but which would fail to capture the entire paradigm in (162): (162a) would be predicted to be consistent, for the reasons given in section 6.1.

On the other hand, the pattern in (162) can be understood on the Exclusion account. The Exlusion theory only involves part-structure insofar as part-structure percolates from the lexical meanings of summative predicates. As such, this theory can be extended to a general theory of exclusion in predication even for predicates without part-quantification. The Exclusion theory as extended to integrative predicates would posit that the predicates

30 The lack of data in (162a-ii) is an accident of the fact that *comedy* and *tragedy* are nouns. With adjectival same-class integrative predicates, we can observe inconsistency when the adjectives are stacked rather than conjoined:

(i) This is a federal #(and) provincial responsibility.

comedy and *tragedy* (163a) are strengthened through local Exh (Pexh) operators (163b), creating contradictions in (162a) but not (162b).

(163) a. $[\![\text{comedy}]\!] = \lambda x.\mathsf{comedy}(x)$.

b. $[\![\text{This [Pred-Exh}_{\text{ALT}}\text{ comedy] is a [Pred-Exh}_{\text{ALT}}\text{ tragedy]}]\!]$

$$= 1 \text{ iff this} \begin{pmatrix} \text{comedy \&} \\ \text{not tragedy \&} \\ \text{not epic} \end{pmatrix} \text{is a} \begin{pmatrix} \text{tragedy \&} \\ \text{not comedy \&} \\ \text{not epic} \end{pmatrix}.$$

Of the theories of homogeneity reviewed here, only the Exclusion theory can collapse the co-predication pattern touching on the *quantificational* strength of summative predicates (157) and the *conceptual* strength of integrative predicates (162).

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